MODERN ALGEBRA
SPRING 2025

ASSIGNMENT 13.1
DUE APRIL 30

Exercise 1. Let A be a finite abelian group.

- **a.** If p is prime and A is a p-group of order p^e, use Theorem 1 of Handout 2 to show that for each $0 \leq f \leq e$, A has a subgroup of order p^f.
- **b.** If A has order n and d is a positive integer dividing n, use part **a** and Theorem 2 of Handout 1 to show that A has a both a subgroup and a quotient of order d. [*Remark.* The existence of the quotient follows from the existence of the subgroup. Why?]
- **c.** How does the result of part **b** differ from the corresponding result for finite cyclic groups?

Exercise 2. Classify (up to isomorphism) the abelian groups of order $n = 2906631$. [*Remark.* There are 18 isomorphism classes.]