



MODERN ALGEBRA  
SPRING 2025

ASSIGNMENT 2.2  
DUE FEBRUARY 5

**Exercise 1.** Construct Cayley tables for the dihedral groups  $D_3$  and  $D_4$ .

**Exercise 2.** Given  $n \geq 3$ , let  $P \subset \mathbb{R}^2$  be a regular  $n$ -gon. Give each vertex of  $P$  a unique label (in any way you like). Let  $r : P \rightarrow P$  denote the smallest possible counterclockwise rotation of  $P$  about its center, and let  $f : P \rightarrow P$  denote any symmetry of  $P$  that reverses the ordering of the labels of the vertices of  $P$ .

- a. Explain why  $D_n = \{\text{id}, r, r^2, \dots, r^{n-1}, f, rf, r^2f, \dots, r^{n-1}f\}$ .
- b. Show that  $fr = r^{n-1}f$ .
- c. Show that  $D_n$  is nonabelian by proving that  $rf \neq fr$ .

**Exercise 3.** How large is the group of symmetries of a regular tetrahedron? Is it abelian?