MODERN ALGEBRA
SPRING 2025

ASSIGNMENT 3.2
DUE FEBRUARY 12

Exercise 1. Let G and H be groups. If $g \in G$ and $h \in H$ both have finite order, show that the order of (g, h) in $G \times H$ is the least common multiple of the orders of g and h.

Exercise 2. Lang, exercise II.1.13.

Exercise 3. If G is a group and $\{H_i \,|\, i \in I\}$ is a collection of subgroups of G, prove that

$$\bigcap_{i \in I} H_i$$

is a subgroup of G. That is, an (arbitrary) intersection of subgroups of G is again a subgroup of G.

Exercise 4. Show that the subset R of rotations in D_n is a subgroup of D_n. Which subgroups of D_n contain only the identity and flips?

Exercise 5. Let G be a group and let $H \subseteq G$. Suppose H is finite. Prove that $H < G$ if and only if $H \neq \varnothing$ and $xy \in H$ for all $x, y \in H$. In other words, a nonempty *finite* subset of a group is a subgroup if and only if it is closed under that group's binary operation. [*Suggestion.* If H is closed under the operation on G, then for any $x \in H$ the left translation map $\lambda_x(y) = xy$ maps H injectively to itself. Since H is finite, λ_x must therefore also be surjective. Use this to prove the existence of an identity and inverses in H.]