Modern Algebra
Spring 2025

Assignment 4.2
Due February 19

Exercise 1. Textbook exercise II.1.19.

Exercise 2. Prove that every subgroup of a cyclic group (finite or infinite) is cyclic.

Exercise 3. Let $n \in \mathbb{N}$. Prove that for every $d \in \mathbb{N}$ dividing n, $\mathbb{Z}_n$ has a unique (necessarily cyclic) subgroup of order d. [*Suggestion.* Show that the elements of order d in $\mathbb{Z}_n$ have the form $k\frac{n}{d}$ with $k \in \mathbb{Z}$, and therefore all belong to $\langle n/d \rangle$.]

Exercise 4. Let $n \in \mathbb{N}$. Prove that $R_n : \mathbb{Z} \to \mathbb{Z}_n$ is a homomorphism. [*Suggestion.* Use the definition of $\oplus$ in $\mathbb{Z}_n$ and the properties of R_n established in Exercise 1.2.1.]