Modern Algebra | Assignment 5.1
Spring 2025 | Due February 26

Exercise 1. Let $(A, +)$ be an abelian group and let $n \in \mathbb{Z}$. Define $f : A \to A$ by $f(a) = na$. Prove that f is an endomorphism. Where does your argument require the hypothesis that A is abelian?

Exercise 2. With A and n as above, prove that if A is finite and $\gcd(|A|, n) = 1$, then f is an isomorphism. In other words, for any $a \in A$ the equation $nx = a$ has a unique solution $x \in A$.

Exercise 3. Lang, Exercise II.3.3.

Exercise 4. Let $f : \mathbb{C}^\times \to \mathbb{C}^\times$ be given by $f(z) = z/|z|$.

- **a.** Show that f is a homomorphism.
- **b.** Explicitly describe $\operatorname{im} f$ and $\ker f$.

Exercise 5. Let $f : G \to H$ be a homomorphism of groups and let $x \in G$.

- **a.** Show that the order of $f(x)$ divides the order of x.
- **b.** If f is a monomorphism, prove that the order of $f(x)$ *equals* the order of x.