Modern Algebra
Spring 2025

Assignment 5.2
Due February 26

Exercise 1. Let $m, n \in \mathbb{Z}$. Prove that $n\mathbb{Z} \subseteq m\mathbb{Z}$ if and only if $m|n$. So containment between subgroups of $\mathbb{Z}$ corresponds to divisibility in $\mathbb{Z}$.

Exercise 2. Let S and T be sets with $|S| = |T|$.

a. Prove that $\mathrm{Perm}(S) \cong \mathrm{Perm}(T)$. So, up to isomorphism, $\mathrm{Perm}(S)$ only depends on $|S|$. [*Suggestion.* Let $f : S \to T$ be any bijection, and show that the map $\mathrm{Perm}(S) \to \mathrm{Perm}(T)$ by $\sigma \mapsto f \circ \sigma \circ f^{-1}$ is an isomorphism.]

b. Conclude that if $|S| = n \in \mathbb{N}$, then $\mathrm{Perm}(S) \cong S_n$.