



MODERN ALGEBRA  
SPRING 2025

ASSIGNMENT 8.2  
DUE MARCH 26

**Exercise 1.** Let  $n \in \mathbb{N}$  and set

$$D(n) = \{d \in \mathbb{N} \mid d \text{ divides } n\}.$$

- a.** Prove that the function  $\sigma : D(n) \rightarrow D(n)$  given by  $\sigma(d) = n/d$  is a bijection. [*Suggestion.* It suffices to show that  $\sigma$  is injective. Why?]
- b.** Use part **a** to show that if  $X = \{x_d \mid d \in D(n)\}$  is a set indexed by  $D(n)$ , then  $X = \{x_{n/d} \mid d \in D(n)\}$ .

**Exercise 2.** Lang, II.5.3

**Exercise 3.** Lang, II.5.5

**Exercise 4.** Lang, II.6.2