



MODERN ALGEBRA  
SPRING 2025

ASSIGNMENT 9.1  
DUE APRIL 1

**Exercise 1.** Draw the directed graphs of each permutation in Exercise 8.3.1. Indicate the disjoint cycles in each.

**Exercise 2.** It is not hard to show that

$$S_3 = \{(1), (12), (13), (23), (123), (132)\} = \langle (12), (123) \rangle.$$

- a. Draw the directed graphs of the left translations of  $S_3$  by  $(12)$  and  $(123)$ .
- b. Use part a to draw the Cayley graph of  $S_3$  relative to the generators  $(12)$  and  $(123)$ .

**Exercise 3.** Let  $(i_1 i_2 \cdots i_r)$  be an  $r$ -cycle in  $S_n$  and let  $\sigma \in S_n$  be arbitrary. Prove that

$$\sigma(i_1 i_2 \cdots i_r)\sigma^{-1} = (\sigma(i_1) \sigma(i_2) \cdots \sigma(i_r)),$$

directly or with directed graphs.

**Exercise 4.** Use the preceding exercise to show that any two  $r$ -cycles in  $S_n$  are conjugate. [Warning. This is not as trivial as it may appear: there's a subtlety that's easy to overlook. Be careful!]