1. For each of the following, find the limit, it if exits. If the limit does not exist, explain why it does not.

   (a) $\lim\limits_{x \to \sqrt{3}} \dfrac{x^2 - 3}{x - \sqrt{3}}$

   (b) $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{2x}$

   (c) $\lim\limits_{x \to \infty} (\ln x)^{\frac{1}{x}}$

2. For each of the following, find the limit, if it exists. If the limit does not exist, explain why it does not exist.

   (a) $\lim\limits_{x \to 2} \dfrac{|x - 2|}{x - 2}$

(b) $\lim_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

3. For each of the following, find the indicated derivative.

(a) $f(x) = 3\sqrt{x} - \dfrac{1}{x} + 4$

(b) $y = \dfrac{\cos x}{1 - x}$, find $\dfrac{dy}{dx}$

(c) $y = 3\cos(2x)$, find $\dfrac{d^2y}{dx^2}$

4. For each of the following, find the indicated derivative.

(a) $xy^2 - y = x^2$, find $\dfrac{dy}{dx}$ when $x = 1$

(b) $h(x) = f(g(3x))$, find $h(2)$ if $g(6) = 1$, $f(1) = 4$, $f'(1) = \frac{1}{2}$, and $g'(6) = 3$

5. (a) If $\int_0^k (2kx - x^2)\,dx = 18$, find k

(b) Find all antiderivatives (or the indefinite integral) of $5\sin(3x)$

(c) Find all antiderivatives (or the indefinite integral) of $\dfrac{x}{\sqrt{3x^2 + 5}}$

6. (a) Find all the antiderivatives (or indefinite integral) of $\dfrac{5x}{2x^2 - 1}$

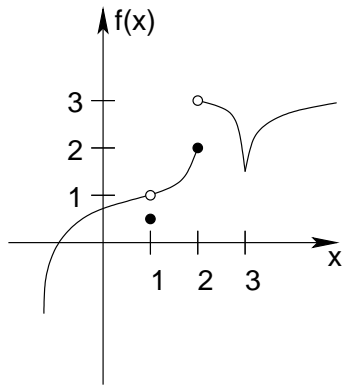(b) Find all antiderivatives (or the indefinite integral) of $(3x - 5)^6$

(c) If $\int_1^5 f(x)\, dx = 4$ and $\int_3^5 f(x)\, dx = -7$, then what is $\int_1^3 f(x)\, dx$?

7. (a)

$$f(x) = \begin{cases} 3x + 1 & \text{if } x \leq 2 \\ cx^2 & \text{if } x > 2 \end{cases}$$

Is there a value of c for which f is continuous on $(-\infty, \infty)$? If so, find it. If not, explain why there is no such c.

(b)

i. For which values of x is f discontinuous?

ii. For which values of x is f <u>not</u> discontinuous?

8. (a) Find $\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$ if $f(x) = x^3 - 4$.

(b) Find $\lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$ if $f(x) = x^3 - 4$.

9. Let $F(x) = \int_0^x \sqrt{t^3 - 1} \, dt$.

(a) Find $F'(x)$.

(b) Find $\lim\limits_{x \to 0} F(x)$.

10. A function f that is continuous for all real numbers x has $f(3) = -1$ and $f(7) = 1$. If $f(x) = 0$ for exactly one value of x, then which of the following could be x? Justify your answer.
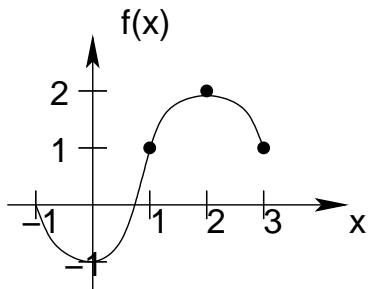
a) -1   b) 0   c)1   d) 4   e) 9

11. If $f'(x) = x^2 + x - 12$, then f is increasing on

(a) $(-4, 3)$

(b) $(-3, 4)$

(c) $\left(-\infty, -\frac{1}{3}\right)$

(d) $(-\infty, -4)$ and $(3, \infty)$

(e) None of the above.

Justify your answer.

12. Suppose that $f(1) = 0$ and that $1 \le f'(x) \le 2$ for x in $[0, 4]$. Use the Mean Value Theorem to explain why $f(4)$ cannot be 10.

13.

(a) What is the average rate of change of f on $[-1, 3]$?

(b) On what intervals is $f'(x)$ increasing?

(c) On what intervals is $f'(x)$ decreasing?

14. The volume of a cylindrical tin can with a top and bottom is to be 18π cubic inches. If a minimal amount of tin is to be used to construct the can, what must be the height, in inches, of the can? (You may want to know that the surface area of a cylinder, excluding a top and bottom, is $2\pi rh$.).

15. The sides of the rectangle increase in such a way that $\dfrac{dz}{dt} = \dfrac{1}{2}$ and $\dfrac{dx}{dt} = 2\dfrac{dy}{dt}$. At the instant when $x = 3$ and $y = 2$, what is the value of $\dfrac{dx}{dt}$?