Math 1311
Test 1
Fall 2004
SOLUTIONS

1. Apply the limit laws to evaluate the following limits or show how the indicated limit does not exist.

   (a) (10 points)

$$\begin{aligned}\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7} &= \lim_{x\to 7}\frac{(\sqrt{x+2}-3)(\sqrt{x+2}+3)}{(x-7)(\sqrt{x+2}+3)}\\ &= \lim_{x\to 7}\frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}\\ &= \lim_{x\to 7}\frac{(x-7)}{(x-7)}\cdot\frac{1}{(\sqrt{x+2}+3)}\\ &= \frac{1}{\sqrt{7+2}+3}\\ &= \frac{1}{3+3}\\ &= \frac{1}{6}\end{aligned}$$

   (b) (10 points)

$$\begin{aligned}\lim_{x\to 1^+}\frac{1-x}{|1-x|} &= \lim_{x\to 1^+}\frac{1-x}{-(1-x)}\\ &= -1\end{aligned}$$

2. Apply the limit laws to evaluate the following limits or show how the indicated limit does not exist.

   (a) (10 points)

$$\begin{aligned}\lim_{x\to 0}\frac{\tan 2x}{\tan 3x} &= \lim_{x\to 0}\frac{\sin 2x}{\cos 2x}\cdot\frac{\cos 3x}{\sin 3x}\\ &= \lim_{x\to 0}\frac{\frac{\sin 2x}{2x}\cdot 2x}{\frac{\sin 3x}{3x}\cdot 3x}\cdot\lim_{x\to 0}\frac{\cos 3x}{\cos 2x}\\ &= \frac{2}{3}\cdot 1\cdot 1\\ &= \frac{2}{3}\end{aligned}$$

(b) (10 points)

$$\begin{aligned}
\lim_{x\to 0}\frac{1-\cos 3x}{2x^2} &= \lim_{x\to 0}\frac{(1-\cos 3x)(1+\cos^2 3x)}{2x^2(1+\cos 3x)}\\
&= \lim_{x\to 0}\frac{1-\cos^2 3x}{2x^2(1+\cos 3x)}\\
&= \lim_{x\to 0}\frac{\sin^2 3x}{2x^2(1+\cos 3x)}\\
&= \lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\left[\frac{9x^2}{2x^2}\right]\lim_{x\to 0}\frac{1}{1+\cos 3x}\\
&= \frac{9}{4}
\end{aligned}$$

3. Apply the definition of the derivative to find $f'(x)$ for

(a) (10 points) $f(x) = 3 - 2x^2$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{3-2(x+h)^2-3+2x^2}{h}\\
&= \lim_{h\to 0}\frac{3-2x^2-4xh-h^2-3+2x^2}{h}\\
&= \lim_{h\to 0}\frac{-h(4x+h)}{h}\\
&= \lim_{h\to 0}-(4x+h)\\
&= -4x
\end{aligned}$$

(b) (10 points) $f(x) = \frac{1}{3-x}$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{\dfrac{1}{3-(x+h)}-\dfrac{1}{3-x}}{h}\\
&= \lim_{h\to 0}\frac{\dfrac{1}{3-(x+h)}-\dfrac{1}{3-x}}{h}\cdot\frac{((3-x)(3-(x+h))}{(3-x)(3-(x+h))}\\
&= \lim_{h\to 0}\frac{3-x-(3-x-h)}{h(3-x)(3-x-h)}\\
&= \lim_{h\to 0}\frac{h}{h(3-x)(3-x-h)}\\
&= \frac{1}{(3-x)^2}
\end{aligned}$$

4. A population of chipmunks moves into a new region at time $t = 0$. At time t (in months), the population numbers

$$P(t) = 100[1 + (0.3)t + (0.04)t^2].$$

(a) (10 points) How long does it take for this population to double its initial size $P(0)$?

$$\begin{aligned} P(0) &= 100[1 + 0 + 0] = 100 \\ P(t) &= 200 = 100 + 30t + 4t^2 \\ 4t^2 &+ 30t - 100 = 0 \\ 2t^2 &+ 15t - 50 = 0 \\ (2t &- 5)(t + 10) = 0 \\ t &= \frac{5}{2} \quad \text{or} \quad \cancel{t = -10} \end{aligned}$$

(b) (10 points) What is the rate of growth of the population when $P = 200$?

$$P'(t) = 0 + 30 + 8t$$

Notice that $P = 200$ when $t = \dfrac{5}{2}$

$$\begin{aligned} P'(200) &= 30 + 8\left(\frac{5}{2}\right) \\ &= 30 + 20 \\ &= 50 \end{aligned}$$

5. Find the derivative $f'(x)$ by applying the differentian rules.

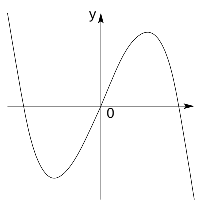
(a) (10 points)

$$\begin{aligned} f(x) &= \frac{2x^3 - 3x^2 + 4x - 5}{x^2} \\ &= 2x - 3 + \frac{4}{x} - \frac{5}{x^2} \\ &= 2x - 3 + 4x^{-1} - 5x^{-2} \\ f'(x) &= 2 - 0 - \frac{4}{x^2} + \frac{10}{x^3} \\ f'(x) &= 2 - \frac{4}{x^2} + \frac{10}{x^3} \end{aligned}$$

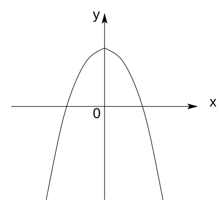
(b) (10 points)

$$
\begin{aligned}
f(x) &= \frac{x^3 - 4x + 5}{x^2 + 9} \\
f'(x) &= \frac{(x^2+9)(3x^2-4) - (x^3-4x+5)(2x)}{(x^2+9)^2} \\
&= \frac{3x^4 + 23x^2 - 36 - 2x^4 + 8x^2 - 10x}{(x^2+9)^2} \\
&= \frac{x^4 + 31x^2 - 10x - 36}{(x^2+9)^2}
\end{aligned}
$$

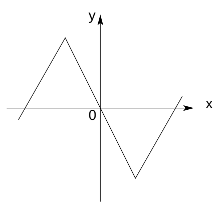
6. (5 points each) Match the graph of each function in (a)-(d) with the graph of its derivative in I-IV. Give reasons for your choice.



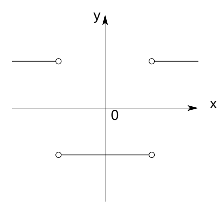
(a)



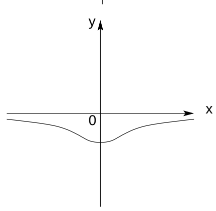
II



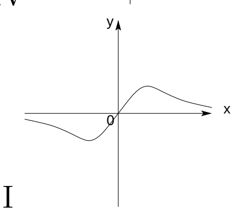
(b)



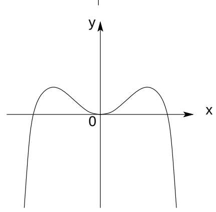
IV



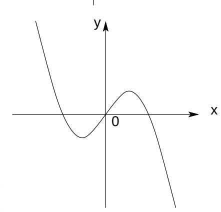
(c)



I



(d)



III