Name: _____

Math 1311
Test 3
Fall 2004

1. Solve the initial value problem in $\dfrac{dy}{dx} = \dfrac{1}{\sqrt[3]{x}}$; $y(1) = 1$.

2. First calculate (in terms of n) the sum

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

to approximate the area A of the region under $f(x) = x^3$ above the interval $[0, 3]$. Then find A exactly by taking the limit as $n \to \infty$.

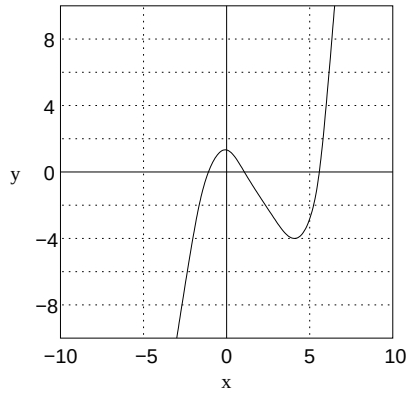
3. Find the limit of $\lim_{x \to \pi} \frac{1 + \cos x}{(x - \pi)^2}$.

4. Find the limit of $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{1 - \cos x} \right)$.

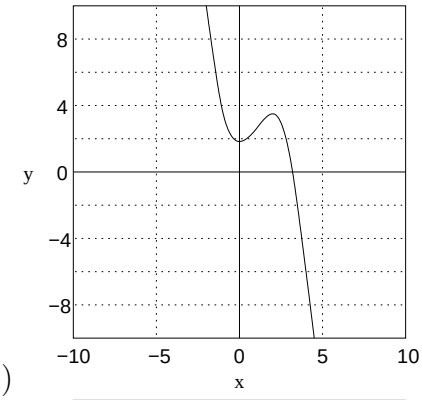
5. Sketch the graph of $f(x) = \dfrac{x^3}{x^2 - 1}$ indicating all critial points, inflection points, and asymptotes. Show the concave structure clearly.

6. Sketch the graph of $f(x) = x^4 - 12x^2$ indicating all critial points, inflection points, and asymptotes. Show the concave structure clearly.

7. Match the graph of each function in (a)-(f) with the graph of its second derivative in I-VI.



(a)



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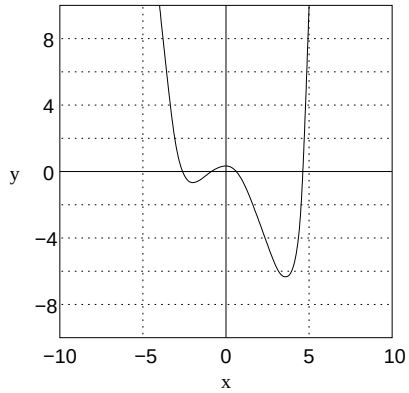
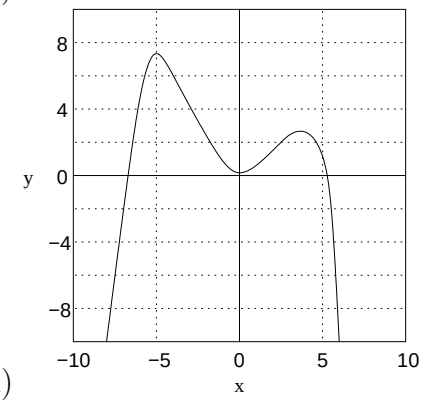
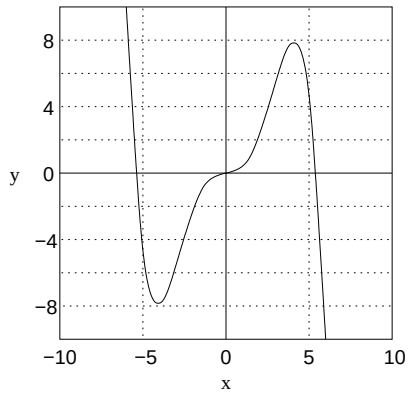


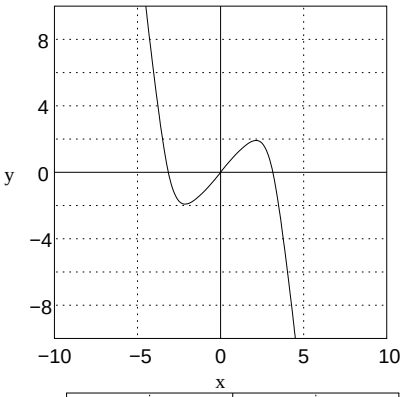
(c)



(d)



(e)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4
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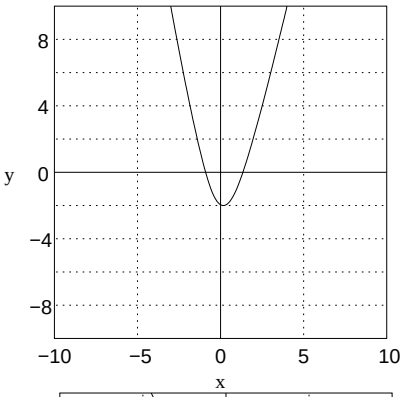
−4
−8
−10 −5 0 5 10
x

I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4
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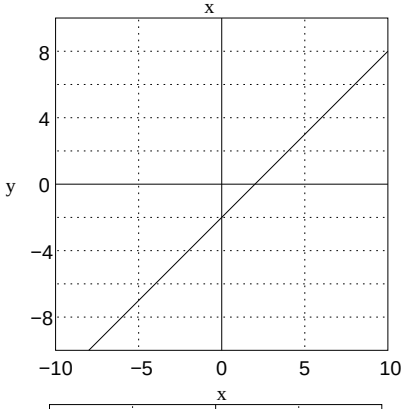
−4
−8
−10 −5 0 5 10
x

II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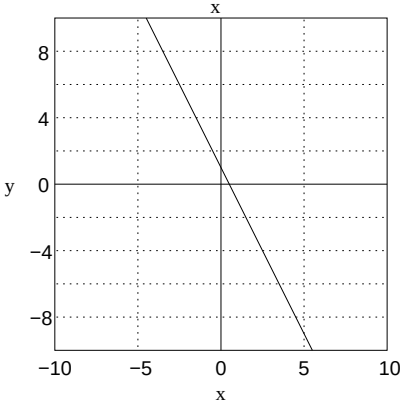
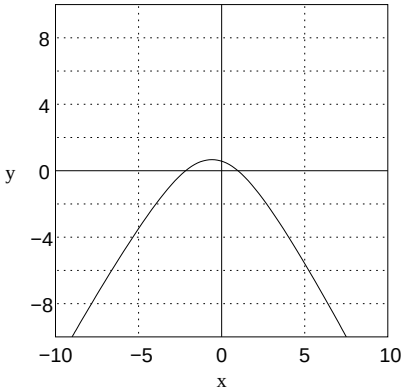
4
y 0
−4
−8
−10 −5 0 5 10
x

III


8
4
y 0
−4
−8
−10 −5 0 5 10
x

IV


8
4
y 0
−4
−8
−10 −5 0 5 10
x

V