Name:____________________

**Math 1311**
**Fall 2004**
**PreTest III**

1. (a) Calculate $dy/dx$ and $d^2y/dx^2$, assuming that $y$ is defined implicitly as a function of $x$ by $\sin^2 x + \cos^2 y = 1$.

   (b) Apply the second derivative test to find the local maxima and local minima of $f(x) = x^3(x+2)^2$ and apply the inflection point test to find all inflection points.

2. Sketch the graph of $f(x) = x^{1/3}(6-x)^{2/3}$ indicating all critical points and inflection points. Apply the second derivative test at each critical point. Show the correct concave structure and indicate the behavoir of $f(x)$ as $x \to \pm\infty$.

3. Begin with a calcular-generated graph of the curve $f(x) = \dfrac{x^5 - 4x^2 + 1}{2x^4 - 3x + 2}$. Then use a calculator to locate accurately the vertical asymptotes and the critical and inflection points of $f(x)$. Finally, use a calculator to produce graphs that display the major features of the curve, including any vertical, horizontal, and slant asymptotes.

4. Find the limits of the following.

   (a) $\displaystyle\lim_{x\to\infty} \frac{\ln(\ln x)}{x \ln x}$

   (b) $\displaystyle\lim_{x\to\frac{1}{2}} \frac{2x - \sin \pi x}{4x^2 - 1}$

5. (a) For $f(x) = \left(1 + \dfrac{1}{x^2}\right)^x$

      i. First use your own calculator to graph the function $f(x)$ with an $x$-range sufficient to suggest its behavoir both as $x \to 0^+$ and as $x \to +\infty$.

      ii. Then apply l'Hôpital's rule as necessary to verify this suspected behavoir near zero and $+\infty$.

      iii. Finally, estimate graphically and/or numerically the maximum value attained by $f(x)$ for $x \geq 0$. If possible, find this maximum value exactly.

   (b) Find the limit of $\displaystyle\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.

6. (a) A particle moves along the $x$-axis with the given acceleration function $a(t) = 10 - 30t$, initial position $x(0) = 5$, and initial velocity $v(0) = -5$. Find the particle's position function $x(t)$.

(b) First calculate (in terms of $n$) the sum

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

to approximate the area $A$ of the region under $f(x) = 9 - x^2$ above the interval $[0, 3]$. Then find $A$ exactly by taking the limit as $n \to \infty$.