Name:_____

Math 1311
Spring 2004
Final Exam

Please write clear, complete solutions with all work shown where required. No credit will be given to unsubstantiated results. Be neat and orderly. Each solution is worth 10 points.

1. In each case, find the limit if it exists. If it doesn't, explain why.

   (a) $\lim_{x\to\infty} \dfrac{x-2}{|x-2|}$

   (b) $\lim_{x\to 0} \dfrac{\sin 3\pi x}{\sin 2\pi x}$

2. In each case, find the value of the limit if it exists. If it doesn't, explain why.

   (a) $\lim_{h\to 0} \dfrac{f(0.5+h)-f(0.5)}{h}$ where $f(x)=\cos \pi x$

   (b) $\lim_{h\to 0} \dfrac{\sqrt{a+h}-\sqrt{a}}{h}$

3. Find each of the following:

(a) $f'(x)$ if $f(x) = \dfrac{5x^2 + 3x + 1}{x^3}$

(b) $f'(x)$ if $f(x) = \dfrac{2x^3 + 1}{\cos x}$

4. Find each of the following:

   (a) $\frac{dy}{dx}$ when $x = 0$, where y is a function defined implicitly by $x^2 - y = 3xy^2 - 1$

   (b) $h'(1)$ where $h(x) = f(g(x))$, $g(1) = 3$, $f'(3) = 4$, $f'(1) = 6$, and $g'(0) = 3$

5. Find each of the following

   (a) All antiderivatives of $(5x - 7)^{10}$

   (b) $\int \frac{2x - 1}{(2x^2 - 2x + 8)^4} dx$

6. Find each of the following:

(a) $\int 3x\sqrt{x^2+1}dx$

(b) $\int_0^{\pi} \cos^3 x \sin x dx$

7. Sketch the region under the curve $y = 4 - x^2$ and the x-axis, showing the polygon corresponding to any Riemann sum method. Find a formula for the Riemann sum and then find the area under the curve by taking a limit.

8. Suppose that the water is being emptied from a spherical tank of radius $8ft$. If the radius of the water in the tank is $6ft$ and is decreasing at the rate of $2ft/s$, at what rate is the depth$h$ of the the water in the tank decreasing?

9. There is a 16-foot ladder leaning against a vertical wall. The tip of the ladder is sliding down the wall at the rate of 5.6 feet per second. What is the rate of change, in radians, per second, of the angle θ at the instant when the tip of the ladder is 7.0 feet above the ground?

10. Let f be a function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

(a) On what intervals is f increasing?

(b) On what intervals is the graph of f concave downward?

(c) Find the value of k for which f has 11 as its relative minimum.