

Name: _____

Math 1312
Spring 2005
Final

1. Under what conditions on a , b , c does the system

$$3x_1 + x_2 - 3x_3 = a$$

$$x_1 + x_2 + x_3 = b$$

$$5x_1 + 6x_2 + 8x_3 = c$$

have (a) a unique solution; (b) no solution; (c) infinitely many solutions.

2. Find an equation of the plane through $P(3, 3, 1)$ that is perpendicular to the planes $x + y = 2z$ and $2x + z = 10$.

3. Find the area of the triangle with vertices $P(1, 1, 0)$, $Q(1, 0, 1)$, and $R(0, 1, 1)$.

4. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \left(\frac{\ln n}{n} \right)^n$$

5. Determine whether the infinite series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$$

6. Find the Taylor series of the given function at the indicated point a .

$$f(x) = \ln x, \quad a = 1$$

7. Determine whether or not the sequence $\{a_n\}$ converges, and find its limit if it does converge.

$$a_n = \left(\frac{2 - n^2}{3 + n^2} \right)^n$$

8. Solve the initial value problem.

$$9y'' + 42y' + 49y = 0; \quad y(0) = 6, \quad y'(0) = -11$$

9. Evaluate the integral.

$$\int \sin^2 \theta \cos^3 \theta \, d\theta$$

10. Compute the integral.

$$\int e^{-3x} \sin 4x \, dx$$