Math 1312
Spring 2005
Pre-Final

1. Determine for what values of k each system has (a) a unique solution; (b) no solution; (c) infinitely many solutions.

$$3x + 2y = 1$$
$$7x + 5y = k$$

2. Find an equation for the plane that passes through the point $P(1, 3, -2)$ and contains the line of intersection of the planes $x - y + z = 1$ and $x + y - z = 1$.

3. Find the volume of the parallelepiped with adjacent edges $\overrightarrow{OP}$, $\overrightarrow{OQ}$, and $\overrightarrow{OR}$, where $P(1, 1, 0)$, $Q(1, 0, 1)$, and $R(0, 1, 1)$.

4. Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^{3n}}{7^n}$$

5. Determine whether the infinite series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{3 + 5^n}$$

6. Find the Taylor series of the given function at the indicated point a.

$$f(x) = \cos x, \quad a = \frac{\pi}{2}$$

7. Determine whether or not the sequence $\{a_n\}$ converges, and find its limit if it does converge.

$$a_n = \left(1 - \frac{2}{n^2}\right)^n$$

8. Solve the initial value problem.

$$y'' - 2y' - 35y = 0; \quad y(0) = 12, \quad y'(0) = 0$$

9. Evaluate the integral.

$$\int \sin^2 3\alpha \cos^2 3\alpha \, d\alpha$$

10. Use integration by parts to compute the integral.

$$\int x^2 \cos 4x \, dx$$