1. Evaluate the integral
\[ \int \cos^2 x \sin^3 x \, dx \]

2. Evaluate the integral
\[ \int u^2 \sin \left( u^\frac{3}{2} \right) \, du \]

3. Evaluate the integral
\[ \int x^2 \sqrt{1 - x^2} \, dx \]
4. Solve the differential equation
   \[ x \frac{dy}{dx} = 3y + x^4 \cos x, \quad y \left( \frac{\pi}{2} \right) = 0 \]

5. Solve
   \[ y'' - 18y' + 77y = 0, \quad y(0) = 4, \quad y'(0) = 8 \]
6. Evaluate
\[ \int \frac{x^3}{\sqrt{x^2 + 25}} \, dx \]

7. Evaluate
\[ \int \frac{x + 1}{x^3 - x^2} \, dx \]

8. Solve the initial value problem
\[ \frac{dy}{dx} = 2xy^3(2x^2 + 1), \quad y(1) = 1 \]
9. Suppose that a community contains 15000 people who are susceptible to Michaud’s syndrome, a contagious disease. At time $t = 0$ the number $N(t)$ of people who have developed Michaud’s syndrome is 5000 and is increasing at a rate of 500 per day. Assume that $N'(t)$ is proportional to the product of the numbers of those who have caught the disease and those who have not. How long will it take for another 5000 people to develop Michaud’s syndrome?

10. A tank contains 1000 L of a solution consisting of 100 kg of salt dissolved in water. Pure water is pumped into the tank at a rate of 5 L/s, and the mixture—kept uniform by stirring—is pumped out at the same rate. How long will it take until only 10 kg of salt remains in the tank?