Name:____________________

Math 3336
Spring 2005
Final

1. Use Laplace transform to solve the initial value problem.

$$x''(t) + x(t) = f(t), \quad x(0) = x'(0) = 0,$$
$$f(t) = \begin{cases} \sin t & \text{if } 0 \le t \le 2\pi \\ 0 & \text{if } t > 2\pi \end{cases}$$

2. Use Laplace transform to solve the initial value problem

$$x'' + 4x' + 13x = te^{-t}; \quad x(0) = 0, \quad x'(0) = 2$$

3. Find a general solution of the system.

$$\begin{aligned} x_1' &= x_1 + 2x_2 + 2x_3, \\ x_2' &= 2x_1 + 7x_2 + x_3, \\ x_3' &= 2x_1 + x_2 + 7x_3 \end{aligned}$$

4. Find the general solution of the system.

$$\overrightarrow{X}'(t) = A\overrightarrow{X}(t), \quad \overrightarrow{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad A = \begin{pmatrix} 3 & -2 \\ 5 & 1 \end{pmatrix}$$

5. Find the general solution of the system.

$$x' = \begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix} x$$

6. Solve the problem without using Laplace transform.

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

7. Solve the initial value problem without using Laplace transform.

$$y^{(3)} + 10'' + 25y' = 0; \quad y(0) = 3, \quad y'(0) = 4, \quad y''(0) = 5$$

8. Find a basis for the solution space of the following homogeneous linear system.

$$\begin{aligned} x_1 - 3x_2 - 10x_3 + 5x_4 &= 0 \\ x_1 + 4x_2 + 11x_3 - 2x_4 &= 0 \\ x_1 + 3x_2 + 8x_3 - x_4 &= 0 \end{aligned}$$

9. Suppose that A, B, and C are invertible matrices of the same size. Show that the product ABC is invertible and that $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

10. Find the general solution of the differential equation. Primes denote derivatives with respect to x.

$$xy' + 3y = 3x^{-\frac{3}{2}}$$

Table of Laplace Transforms

$f(t)$	$F(s)$	
$e^{at}f(t)$	$F(s-a)$	
$u(t-a)f(t-a)$	$e^{-as}F(s)$	
1	$\frac{1}{s}$	$(s>0)$
t	$\frac{1}{s^2}$	$(s>0)$
t^n $(n \geq 0)$	$\frac{n!}{s^{n+1}}$	$(s>0)$
t^a $(a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}$	$(s>0)$
e^{at}	$\frac{1}{s-a}$	$(s>0)$
$\cos kt$	$\frac{s}{s^2+k^2}$	$(s>0)$
$\sin kt$	$\frac{k}{s^2+k^2}$	$(s>0)$
$\cosh kt$	$\frac{2}{s^2-k^2}$	$(s>\lvert k \rvert)$
$\sinh kt$	$\frac{k}{s^2-k^2}$	$(s>\lvert k \rvert)$
$u(t-a)$	$\frac{e^{-as}}{s}$	$(s>0)$