Math 3336
Spring 2005
Pre-Final

1. Use Laplace transform to solve the initial value problem.

$$x''(t) + 5x'(t) + 4x(t) = f(t), \quad x(0) = x'(0) = 0,$$
$$f(t) = \begin{cases} 1 & \text{if } 0 \le t < 2 \\ 0 & \text{if } t \ge 2 \end{cases}$$

2. Use Laplace transform to solve the initial value problem

$$x'' + 6x' + 18x = \cos 2t; \quad x(0) = 1, \quad x'(0) = -1$$

3. Find the general solution of the system

$$\begin{aligned} x_1' &= 5x_1 + 5x_2 + 2x_3, \\ x_2' &= -6x_1 - 6x_2 - 5x_3, \\ x_3' &= 6x_1 + 6x_2 + 5x_3 \end{aligned}$$

4. Find the general solution of the system.

$$x' = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix} x$$

5. Find the general solution of $x' = AX$.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

6. Solve the initial value problem without using Laplace transform.

$$y'' + 2y' + 2y = \sin 3x; \quad y(0) = 2, \quad y'(0) = 0$$

7. Solve the initial value problem without using Laplace transform.

$$2y^{(3)} - 3y'' - 2y' = 0; \quad y(0) = 1, \quad y'(0) = -1, \quad y''(0) = 3$$

8. Find a basis for the solution space of the following homogeneous linear system.

$$\begin{aligned} x_1 + 3x_2 + 4x_3 &= 0 \\ 3x_1 + 8x_2 + 7x_3 &= 0 \end{aligned}$$

9. Show that every invertible matrix is a product of elementary matrices.

10. Find the general solution of the differential equation. Primes denote derivatives with respect to x.

$$(x^2 - 1)y' + (x - 1)y = 1$$