Name	: <u> </u>
	Math 3336 Sprint 2005 Test I
Show	all your steps.
1.	Determine the values of $a$ so that the following system in the unknowns $x, y, z$ has:
	x + y - z = 1 $2x + 3y + az = 3$ $x + ay + 3z = 2$
	(a) no solution,
	(b) more than one solution,
	(c) a unique solution:

2. Consider the following matrix

$$A = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}$$

(a) Reduce A to a row-echelon form

(b) Reduce A to a reduced row-echelon form

(c) If  $A^{-1}$  exists, find it.

3. (a) Determine whether or not the set W of all vectors in  $\mathbb{R}^4$  is a subspace of  $\mathbb{R}^4$ , where

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} : x_1 + x_2 = x_3 + x_4 \right\}$$

(b) Suppose A is an  $n \times n$  matrix and that k is a constant scalar. Show that the set of all vectors  $\vec{x}$  such that  $A\vec{x} = k\vec{x}$  is a subspace of  $\mathbb{R}^4$ .

4. Solve the differential equation

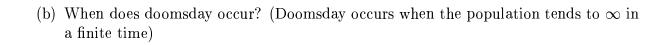
$$xy' = 3y + x^4 \cos x, \quad y\left(\frac{\pi}{2}\right) = 0$$

5.	A tank initially contains 20 gallons of pure water. Brine containing 2 pounds of salt per
	gallon enters the tank at 3 gal/min and the perfectly mixed solution leaves the tank at 4
	gal/min.

(a) Find the amount of salt in the tank after t time.

(b) Find the amount of salt in the tank after 15 minutes.

- 6. Consider an animal population P(t) with constant death rate  $\delta = 0.01$  (deaths per animal per month) and with birth rate  $\beta$  proportional to P. Suppose that P(0) = 200, and P'(0) = 2.
  - (a) When P = 1000?



7. (a) Find the critical points (equilibrium points/constant solutions) of the differential equation

$$\frac{dx}{dt} = x^2 - 7x + 10.$$

(b) Determine the stability of the critical points without solving the equation.

(c) Draw a rough graph of the solution curves.