Name:_____

Math 3336
Sprint 2005
Test I

Show all your steps.

1. Determine the values of a so that the following system in the unknowns x, y, z has:

$$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + az &= 3 \\ x + ay + 3z &= 2 \end{aligned}$$

(a) no solution,

(b) more than one solution,

(c) a unique solution:

2. Consider the following matrix

$$A = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}$$

(a) Reduce A to a row-echelon form

(b) Reduce A to a reduced row-echelon form

(c) If A^{-1} exists, find it.

3. (a) Determine whether or not the set W of all vectors in $\mathbb{R}^4$ is a subspace of $\mathbb{R}^4$, where

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} : x_1 + x_2 = x_3 + x_4 \right\}$$

(b) Suppose A is an $n \times n$ matrix and that k is a constant scalar. Show that the set of all vectors $\vec{x}$ such that $A\vec{x} = k\vec{x}$ is a subspace of $\mathbb{R}^4$.

4. Solve the differential equation

$$xy' = 3y + x^4 \cos x, \quad y\left(\frac{\pi}{2}\right) = 0$$

5. A tank initially contains 20 gallons of pure water. Brine containing 2 pounds of salt per gallon enters the tank at 3 gal/min and the perfectly mixed solution leaves the tank at 4 gal/min.

   (a) Find the amount of salt in the tank after t time.

   (b) Find the amount of salt in the tank after 15 minutes.

6. Consider an animal population $P(t)$ with constant death rate $\delta = 0.01$ (deaths per animal per month) and with birth rate β proportional to P. Suppose that $P(0) = 200$, and $P'(0) = 2$.

   (a) When $P = 1000$?

(b) When does doomsday occur? (Doomsday occurs when the population tends to ∞ in a finite time)

7. (a) Find the critical points (equilibrium points/constant solutions) of the differential equation

$$\frac{dx}{dt} = x^2 - 7x + 10.$$

(b) Determine the stability of the critical points without solving the equation.

(c) Draw a rough graph of the solution curves.