Name:____________________

Math 3336
Sprint 2005
PreTest I

1. Solve the following differential equations.
   (a) $y' + 2xy = 0$
   Answer: $y(x) = ce^{-x^2}$
   (b) $y' = 2y(1-y), \quad y(0) = 3$
   Answer: $y(x) = \dfrac{3}{3 - 2e^{-2t}}$

2. Solve the equations.
   (a) $(x+2)y' - xy = 0, \quad y(0) = 3$
   Answer: $y(x) = \dfrac{12e^x}{x^2 + 4x + 4}$
   (b) $\dfrac{dy}{dt} = 2t^2 + \dfrac{2}{t}y, \quad y(-2) = 4$
   Answer: $y(t) = 2t^3 + 5t^2$

3. Suppose that a community contains 15,000 people who are susceptible to Michaud's syndrome, a contagious disease. At time $t = 0$ the number $N(t)$ of people who have developed Michaud's syndrome is 5000 and is increasing at the rate of 500 per day. Assume that $N'(t)$ is proportional to the product of the numbers of those who have caught the disease and of those who have not. How long will it take for another 5000 people to develop Michaud's syndrome?

4. A cup of hot chocolate is initially 170° F and is left in a room with an ambient temperature of 70° F. Suppose that at time $t = 0$ it is cooling at a rate of 20° F per minute. Assume that Newton's law of cooling applies (the rate of cooling is proportional to the difference between the current temperature and the ambient temperature.)
   (a) Write an initial value problem that models the temperature of the hot chocolate.
   (b) How long does it take the hot chocolate to cool to a temperature of 110° F?

5. Suppose a species of fish in a particular lake has a population that is modeled by the logistic population model with growth rate k, carrying capacity N, and time t measured in years. Adjust the model to account for each of the following situations.
   (a) 100 fish are harvested each year.
   (b) One-third of the fish population is harvested annually.

(c) The number of fish harvested each year is proportional to the square root of the number of fish in the lake.

6. Solve the system

$$\begin{aligned} 2x + y - 2z &= 10 \\ 3x + 2y + 2z &= 1 \\ 5x + 4y + 3z &= 4 \end{aligned}$$

Answer: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

7. What conditions must be placed on a, b, c so that the following system is unknowns x, y, z has

   (i) a unique solution
   Answer: not possible

   (ii) no solution
   Answer: $c + 2b - 5a \neq 0$

   (iii) infinitely many solutions
   Answer: $c + 2b - 5a = 0$

8. Find the inverse of $A = \begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 6 \\ -1 & 5 & 1 \end{pmatrix}$.

   Answer: $\begin{pmatrix} 31/2 & -17/2 & 11 \\ 9/2 & -5/2 & -3 \\ -7 & 4 & 5 \end{pmatrix}$.

9. Suppose that $A^2 = A$, where A is $n \times n$ matrix. Prove that $|A| = 0$ or $|A| = 1$.

10. Let U and V be subspaces of the vector space W. Their sum $U + V$ is the set of all vectors $\overrightarrow{x}$ of the form $\overrightarrow{x} = \overrightarrow{u} + \overrightarrow{v}$, $\overrightarrow{u} \in U$, $\overrightarrow{v} \in V$. Show that $U + V$ is a subspace of W. If U and V are lines through the origin in $\mathbb{R}^3$, what is $U + V$?