Name: ______________________

Math 3359
Take Home FINAL
Spring 2004
Due Friday 5/7/2004

1. Consider the map $Q_c(x) = x^2 + c$, $-2 \le c \le 0$, defined on the interval $[-2, 2]$.

   (a) Discuss the stability of the fixed points and the periodic points of period 2.

   (b) Draw a bifurcation diagram for the map. Estimate the interval in the parameter c for which window 3 appears.

2. Given a continuous map f on the closed interval $[a, b]$ with an orbit x_0, $x_1 = f(x_0)$, $x_2 = f^2(x_0), \ldots$ such that $x_2 < x_0 < x_1$. Prove that f has a point of minimal period 2.

3. Construct a continuous map that has an 8-cycle (of period 8) but no 16-cycle (of period 16). Give all necessary details.

4. Let f be a continuous map on R. Prove that:

(a) If $f(b) = b$ and $x < f(x) < b$ for all $x \in [a, b]$, then $a \in W^s(b)$ (the basin of attraction of b).

(b) If $f(b) = b$ and $b < f(x) < x$ for all $x \in [a, b]$, then $c \in W^s(b)$.

5. Solve the system $\overrightarrow{x}(n+1) = A\overrightarrow{x}(n)$, $\overrightarrow{x}(0) = \binom{1}{0}$ where $A = \begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix}$.