Name:________________________

Math 3360
Fall 2002
Take Home Test II

1. Suppose that for some constant M with $0 < M < 1$, $|s_{n+2} - s_{n+1}| \le M|s_{n+1} - s_n|$, for $n = 1,2,3,\ldots$. Prove that the sequence (s_n) is Cauchy.

2. Find sup s_n, inf s_n, lim sup s_n, lim inf s_n if

   (a) $s_n = \dfrac{(-1)^n}{n+1}, n = 1,2,3,\ldots$

   (b) $s_n = \dfrac{2}{3}\left(1 - \dfrac{1}{10^n}\right), n = 1,2,3,\ldots$

   (c) $s_n = (-1)^n n, n = 1,2,3,\ldots$

3. Use only the definition of limits to prove that

(a) $\lim_{n\to\infty} \frac{\sin n}{n} = 0$

(b) $\lim_{n\to 3^+} \frac{1}{x-3} = +\infty$

4. Consider the sequence (s_n), when $s_n = \sum_{k=1}^{n} \frac{1}{k}$.

(a) Show that $\lim_{n\to\infty} |s_{n+k} - s_n| = 0$ for any given $k \in \mathbb{N}$.

(b) Is (s_n) Cauchy?

5. Assume $x_1 > 0$ and $x_{n+1} = \frac{1}{3 + x_n}$. Prove that the sequence (x_n) converges and compute its limit.