Name: ________________________

Math 3360
Fall 2002
Take Home Test III

Due Wednesday, November 27th, 5:00 p.m.

1. Determine which of the following series converge. Justify your answers.

   (a) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\log n}$

   (b) $\sum_{n=2}^{\infty} \frac{\log n}{n}$

   (c) $\sum_{n=4}^{\infty} \frac{1}{n(\log n)(\log\log n)}$

   (d) $\sum_{n=2}^{\infty} \frac{\log n}{n^2}$

2. Let

   (a) $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ prove that f is discontinuous at $x = 0$ and continuous at every other point

   (i) by using the definition.

   (ii) by verifying the $\in - \delta$ property of theorem 17.2

(b) Prove that $f(x)=\sqrt{x-1}$ is continuous at $x=1$

(i) by the definition.

(ii) by the $\in-\delta$ property.

3. Suppose that f is continuous on $[a,b]$ and $f(x)=0$ on every rational number $x\in[a,b]$. Prove that $f(x)=0$ for every $x\in[a,b]$.

4. Suppose that f is continuous on $[a,b]$ and one-to-one $(\textit{if } x_1\neq x_2 \textit{ then } f(x_1)\neq f(x_2))$. Prove that f is strictly increasing on $[a,b]$.

5. (a) Find the values of x for which the series $\sum_{n=1}^{\infty}\frac{(-1)^1(x-1)^n}{2^x n^2}$ converges.

(b) Determine whether the series converges or diverges $\sum_{n=1}^{\infty}\frac{(-1)^1(\frac{4}{3})^n}{n^4}$.