Due Monday 10/4 in my mailbox before 5:00 p.m.

1. Use the definition of a limit to show that
   (a) \( \lim_{n \to \infty} \frac{\sin n}{n} = 0 \)
   (b) \( \lim_{n \to 3^+} \frac{1}{x - 3} = \infty \)

2. (a) Let \( \{x_n\} \) be a bounded sequence of real numbers and \( \{x_{k_n}\} \) be a monotone subsequence. Prove that \( \{x_{k_n}\} \) converges to a limit.
   (b) If \( \lim_{n \to \infty} x_n = L = 0 \), prove that \( \lim_{n \to \infty} \frac{1}{x_n} = \frac{1}{L} \)

3. Let \( x_{n+1} = \frac{1}{3 + x_n}, \ x_1 > 0 \). Prove that the sequence \( \{x_n\} \) converges and then compute the limit of the sequence.

4. Prove that the function \( f(x) = \frac{1}{x^2} \) is continuous for all real numbers \( x \neq 0 \).

5. Suppose that \( f \) is continuous on \([a, b]\), one-to-one (if \( x_1 \neq x_2 \), then \( f(x_1) \neq f(x_2) \)), and \( f(a) < f(b) \). Prove that \( f \) is monotonically increasing (strictly) on \([a, b]\).

6. Suppose that \( S_1, S_2, \ldots, S_n \) are sets in \( \mathbb{R}^1 \) and that \( S = S_1 \cap S_2 \cap \cdots \cap S_n, S \neq \emptyset \). Let \( B_i = \sup S_i, b_i = \inf S_i, 1 \leq i \leq n \). Find a formula relating \( \sup S \) and \( \inf S \) in terms of the \( \{b_i\} \) and \( \{B_i\} \).

7. Suppose that \( \lim_{x \to a} f(x) = \infty \) and \( \lim_{x \to -\infty} g(x) = a \). Suppose that for some positive number \( M \), we have \( g(x) \neq a \) for \( x < -M \). Prove that \( \lim_{x \to -\infty} f(g(x)) = \infty \).

8. If \( f(x) \) is continuous on \([a, b]\), if \( a < c < d < b \), and \( M = f(c) + f(d) \), prove there exists a number \( \xi \) between \( a \) and \( b \) such that \( M = 2f(\xi) \).

9. Suppose that \( f(x) \) and \( g(x) \) are functions defined for \( x > 0 \), \( \lim_{x \to 0^+} g(x) \) exists and is finite, and \( |f(b) - f(a)| \leq |g(b) - g(a)| \) for all positive real number \( a \) and \( b \). Prove that \( \lim_{x \to 0^+} f(x) \) exists and is finite.
10. Evaluate \( \lim_{n \to \infty} \frac{2 + 2^{\frac{1}{2}} + 2^{\frac{1}{3}} + \cdots + 2^{\frac{1}{n}}}{n} \).

(You need not give a proof but you should show some work or justification. Quote a theorem or what have you. Calculator results or graphical analysis are not acceptable.)