Real Analysis
Test 2
Fall 2004

1. Suppose that f and g are increasing on an interval I and that $f(x) > g(x)$ for all $x \in I$. Denote the inverses of f and g by F and G and their domains by J_1 and J_2, respectively. Prove that $F(x) < G(x)$ for each $x \in J_1 \cap J_2$.

2. (a) $\lim_{x \to \frac{\pi}{2}} \sec x - \tan x$

   (b) $\lim_{x \to 0^+} x^x$

3. Suppose that f and g are uniformly continuous on a subset S of $\mathbb{R}$ (not a closed interval). Prove that the function $h = f - g$ is uniformly continuous.

4. Give examples of

   (i) A bounded sequence that is not Cauchy.

   (ii) A nested sequence $\{I_n\}$ of intervals such that $\bigcap_{n=1}^{\infty} I_n = \emptyset$

   (iii) A continuous but not uniformly continuous function.

   (iv) A Cauchy sequence $\{x_n\}$ and a function f such that $\{f(x_n)\}$ is not Cauchy.

5. Prove <u>directly from the definition</u> that the function $f(x) = x^4$ is uniformly continuous on the closed interval $[0, 1]$. Is f uniformly continuous on $\mathbb{R}$?