Name: ____________________

Math 4364: Theory of Complex Variables
Take Home Test 1
Fall 2003

1. (a) Find all values of z for which $z^5 = -32$.

   (b) Locate these values in the complex plane.

2. (a) Prove that $\eta = e^{-x}(x \sin y - y \cos y)$ is harmonic.

   (b) Find v such that $f(z) = u + iv$ is analytic.

3. (a) Find the image of the semi-infinite strip $x \geq 0$, $0 \leq y \leq \pi$ under the transformation $w = \exp z$, and label corresponding portions of the boundaries.

(b) Sketch the region onto which the sector $r \leq 1$, $0 \leq \theta \leq \frac{\pi}{4}$ is mapped by the transformation $w = z^3$.

4. Suppose that $f(z_0) = g(z_0) = 0$ and that $f'(z_0)$ and $g'(z_0)$ exists, where $g'(z_0) \neq 0$. Show that
$$\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{f'(z_0)}{g'(z_0)}.$$

5. (a) Show that if v and w are harmonic conjugates to u in a domain D, then $v(x, y)$ and $w(x, y)$ can differ at most by an additive constant.

(b) Let $f(z) = u + iv$ be differentiable at a nonzero point $z_0 = r_0 \exp(i\theta_0)$. Show that
$$f'(z_0) = \frac{-i}{z_o}(u_\theta + iv_\theta).$$