## Research Experience for Undergraduates in Mathematics

### Project Summary

**Name of Institution:** Trinity University, San Antonio, Texas

**Major Field:** Mathematics

**Number of Students Involved:** 12 per Academic Year

**Number of Summer weeks on Site:** 7 per Academic Year

**Point of Contact for Student Recruitment:** Professor Scott Chapman

e-mail: schapman@trinity.edu
phone: (210)999-8245
fax: (210)999-8264

**Proposed Web Site Address:** http://www.trinity.edu/departments/mathematics/REU/

**Summary:** In attempting to remain one of the Southwest's premier centers for undergraduate mathematics, the Mathematics Department at Trinity University recognizes the importance of the emerging role of undergraduate research and proposes to reopen our Summer Undergraduate Research Experience in Mathematics Program, which was originally funded by the National Science Foundation during the summers of 1997, 1998 and 1999. Our objectives in asking for three years of new funding for this program are fourfold:

1. To provide student participants with an understanding of, an appreciation for, and an experience in the nature of mathematical research and the life of a mathematical researcher, to a degree which will encourage them to pursue the study of the mathematical sciences on the graduate level.

2. To offer these experiences with a special eye toward students who do not typically have these opportunities. Of particular interest to us are students who are either female or a member of an under-represented minority group.

3. To produce quality mathematical work appropriate for publication in undergraduate research journals and, when possible, in higher level research journals.

4. To gain experience in the use of computers and their interaction in mathematical research.

While we envision the recruiting effort for students to be nationwide, we see, due to our location, a unique opportunity to recruit students at many colleges and universities in South Texas that have traditionally high Hispanic enrollment. We propose an expanded recruiting effort to such schools and a revamping of our past program evaluation efforts. We also propose an expansion of our old program from 4 faculty to 6 faculty and the number of students involved from 6 to 12. The available research areas will include: Abstract Algebra, Difference Equations, Discrete Dynamical Systems, Mathematical Programming, Combinatorics, and Number Theory. Our proposed expansion is in line with Trinity University's strong commitment to undergraduate research, as demonstrated by Trinity's nationally recognized undergraduate research programs in Chemistry, Biology and Computer Science.

# Project Description

## 1 Introduction and Program Overview

The traditional focus of a Bachelor's degree in mathematics has evolved significantly over the past several decades. It was common for a mathematics major in the 1950's or 1960's to be required to complete an upper division course fitting the following description:

> **Theory of Equations** Theory of determinants. Theory, analysis and solution of higher numerical and algebraic equations.[1]

While bits and pieces of the material from such a course have survived in various other mathematics courses, one would be hard pressed to find an institution which today still offers such a course. A similar evolution has occured in the notion of an undergraduate student working on an open ended mathematics problem. While 40 or 50 years ago this would have been almost unimagined, it has today become an important part of many of our mathematics curricula that lead to either a B.S. or a B.A. degree. Nowhere is this more true than at Trinity University, where each undergraduate mathematics major must complete a research project as part of his or her degree requirements.[2]

In attempting to remain one of the Southwest's premier centers for undergraduate mathematics, the Mathematics Department at Trinity University recognizes the importance of the emerging role of undergraduate research and proposes to reopen our Summer Undergraduate Research Experience in Mathematics Program, which was originally funded by the National Science Foundation during the summers of 1997, 1998 and 1999. Our objectives in asking for three years of new funding for this program are fourfold:

1. To provide student participants with an understanding of, an appreciation for, and an experience in the nature of mathematical research and the life of a mathematical researcher, to a degree which will encourage them to pursue the study of the mathematical sciences on the graduate level.

2. To offer these experiences with a special eye toward students who do not typically have these opportunities. Of particular interest to us are students who are either female or a member of an under-represented minority group.

3. To produce quality mathematical work appropriate for publication in undergraduate research journals and, when possible, in higher level research journals.

4. To gain experience in the use of computers and their interaction in mathematical research.

Our earlier grant exhibited many strengths, which we discuss in detail in Section 6 (*Results from Prior REU-NSF Support*). The Department of Mathematics at Trinity offers an atmosphere strong in basic mathematical research, a fact we will address in Section 3 (*The Research Environment*). To be sure, we believe that success in mathematical undergraduate research requires careful mentoring and extreme care in the choice of research topics, areas in which our past program excelled. Our current plans in this direction are explained in detail in Section 2 (Nature of Student Activity).

While we envision the recruiting effort for students to be nationwide, we see, due to our location, a unique opportunity to recruit students at many colleges and universities in South Texas that have a traditionally high enrollment of students of Hispanic ethnicity. We propose an expanded recruiting effort to such schools (see Section 4, *Student Recruitment and Selection*) and a revamping of our past program evaluation efforts (see Section 5, *Evaluation*).

We suggest an expansion of our old program from 4 faculty to 6 faculty and the number of students involved from 6 to 12 (with each faculty member mentoring 2 students). The areas represented along with

[1] *Bulletin of Trinity University*, Vol. LX, Number III, 1963, see description of Math 331.
[2] *Trinity University Courses of Study, 2000-2001*, see description of Math 4194.

participating faculty are as follows: *Scott Chapman*, principal investigator (Abstract Algebra), *Saber Elaydi*, (Difference Equations), *Roberto Hasfura*, (Discrete Dynamical Systems), *Allen Holder* (Mathematical Programming), *Vadim Ponomarenko* (Combinatorics), *Holly Rosson* (Number Theory). The Principal Investigator will act as Program Director and manage all aspects of the grant within the guidelines set forth by NSF and the Trinity University Office of Academic Affairs. Notice that the available research topics are not concentrated in one mathematical area and offer students a wide variety of research options.

Our proposed expansion reflects Trinity University's strong commitment to undergraduate research, as demonstrated by Trinity's nationally recognized undergraduate research programs in Chemistry, Biology and Computer Science. In fact, Trinity University supports undergraduate research in science, mathematics and engineering not only through various sponsored research programs which total more than $1,000,000 per year, but also through restricted Endowments for Undergraduate Research that provide at least $24,000 per year for student use.[3]

We close Section 1 with a timetable of the events surrounding year one of this proposal (2001). The timetables for years 2 and 3 will be based on the year 2001 schedule, with adjustment as determined by the Evaluation process.

| | |
|---|---|
| Week of January 25-29 | Faculty Organizational Meeting |
| February 1 | Program Announcement/Application Form posted on the Trinity Math. Department website and sent directly to target schools in South Texas |
| February 2–26 | Faculty visit target schools in South Texas |
| Week of March 5–9 | Program Director follows up on contacts made in February with potential minority and female applicants |
| Week of March 12–16 | Program Director distributes Alumni Surveys to past Participants |
| March 16 | Application Deadline |
| March 19 | Faculty Meet to Discuss Applications |
| March 20 | Deadline for First Round Offers |
| March 27 | Deadline for Student Responses to First Round Offers<br>Second Round Offers Made (if necessary) |
| April 3 | Deadline for Student Responses to Second Round Offers |
| Week of April 9–13 | Local Information Packets Sent to Students |
| Week of April 23–27 | Campus Housing Arrangements Finalized |
| June 3 | Students Arrive in San Antonio |
| Week of June 4–8 | Opening Week of Program (see daily schedule in Section 2) |
| June 11 | Student and Advisor Finalize Project Topic |
| July 17–18 | Exit Interviews and Evaluation Forms Distributed to Students |
| July 19–20 | Final Oral Project Presentations<br>Students Return Evaluation Forms and Check Out |
| Week of July 23–27 | Faculty Assessment Meetings |
| Weeks of Sept. 3–14 | Preparation of Yearly Progress Report for NSF |

## 2 Nature of Student Activities

Trinity University is located in San Antonio, TX, which is the eighth largest city in The United States. A large part of the city's economic base is provided from tourism, and there are many local attractions for REU participants to enjoy. The Trinity campus is served by a convienient city bus system, and almost every location in San Antonio is accessible to our students, even without an automobile. We believe that a strong recreational component of the program is necessary for success and will routinely schedule weekend activities for the participants. Such activities can vary from a day visiting the city's historic Church Missions, to a visit to the LBJ ranch located in nearby Johnson City.

[3]Information obtained from the Office of the Trinity University Division of Science, Mathematics and Engineering.

Before students arrive for the Program, they will be assigned a Project Director. These decisions will be based on the student's application (in particular, their written statement expressing interest in the program plus two faculty references) as well as a telephone interview of the student by the potential Director. Under normal circumstances, each investigator listed on the grant will direct two students. The students will be assigned shared office space in the Mathematics Department, and whenever possible, students assigned to the same Project Director will share office space. This should facilitate communication between the students and maximize the department's available computing equipment.

During the first five days of the summer program, general topic colloquia will be given by the principal investigators. The purpose of these lectures is to let all of the REU participants become familiar with the projects being worked on that summer and foster collegiality within the group. In the afternoon, the students will meet with their mentors and begin a deeper investigation of their potential Project Topics. This investigation may include a summary of several types of problems from which the student selects their final Topic. An afternoon tea hour is held every afternoon of the project. This is an opportunity for the REU students to meet and interact with other summer research students working on campus in the areas of Chemistry, Physics and Biology. The following schedule will be used for the first week.

| | | | |
|---|---|---|---|
| Monday | 8:00 - 11:00 a.m. | Check into dorm | Witt Residence Hall |
| | 11:00 - 12:00 p.m. | Library Tour | Library |
| | 12:00 - 1:00 p.m. | Lunch | Coates Center |
| | 1:00 - 2:00 p.m. | Orientation | Math Department |
| | 2:00 - 3:30 p.m. | Project Meetings | |
| | 3:30 - 4:30 p.m. | Athletic Facilities Tour | Athletic Center |
| Tuesday | 9:00 - 10:30 a.m. | Topic Colloquium | MMS 104 |
| | 10:30 - 11:00 a.m. | Refreshments | |
| | 11:00 - 12:00 p.m. | Project Meetings | |
| | 12:00 - 1:30 p.m. | Lunch | Coates Center |
| | 1:30 - 2:30 p.m. | Project Meetings | |
| | 2:30 - 3:30 p.m. | Tea hour and conversation | |
| | 3:30 - 5:00 p.m. | Topic Colloquium | MMS 104 |
| Wednesday | 9:00 - 10:30 a.m. | Topic Colloquium | MMS 104 |
| | 10:30 - 11:00 a.m. | Refreshments | |
| | 11:00 - 12:00 p.m. | Project Meetings | |
| | 12:00 - 1:30 p.m. | Lunch | Coates Center |
| | 1:30 - 2:30 p.m. | Project Meetings | |
| | 2:30 - 3:30 p.m. | Tea hour and conversation | |
| | 3:30 - 5:00 p.m. | Project Meetings | |
| Thursday | Same as Tuesday | | |
| Friday | Same as Wednesday | | |

Early in the second week of the program (if not sooner), the student and the Project Director decide formally on a Project Topic. Our past experience indicates a higher rate of success on a project if students work together as a team on a single Project Topic. When appropriate, Project Directors will encourage such collaboration, but the final decision on the Project Topic is made jointly by the student and the Project Director.

The remaining weeks of the Program are devoted to completion of the Project. Regular meetings between the student and the Project Director will be scheduled. Ideally, such meetings will take place no less than three times a week, although this is up to the discretion of the Project Director. The final project has two components: 1) a one hour oral presentation during the last week of the Program, and 2) a written report due before the student leaves campus. The written reports will become part the department's technical report series and will be posted on our web page. Besides being possible preprints of publishable articles, these reports will be useful to prospective and incoming REU students.

Trinity University is fortunate to be near several companies that employ applied mathematicians (The Southwest Research Institute, Motorola, Texaco, The San Antonio Health Sciences Center). Additional guest lectures from applied mathematicians employed by these companies will be scheduled as time permits.

Below are examples of the types of projects available to our REU participants.

**Sets of Lengths and Elasticities in Krull Monoids** by Scott T. Chapman

Let $G$ be an abelian group and $\mathcal{F}(G) = \{\prod_{g \in G} g^{v_g} \mid v_g \in \mathbb{Z}^+ \cup \{0\}\}$ be the multiplicative free Abelian monoid with basis $G$. Given $F \in \mathcal{F}(G)$, we write $F = \prod_{g \in G} g^{v_g(F)}$. The *block monoid* over $G$ (see [21] or [10]) is the set $\mathcal{B}(G) = \{B \in \mathcal{F}(G) \mid \sum_{g \in G} v_g(B) g = 0\}$ supplied with the operation inherited from $\mathcal{F}(G)$. The elements of $\mathcal{B}(G)$ are called blocks. Note that the empty block acts as the identity in $\mathcal{B}(G)$. A block $B$ in $\mathcal{B}(G)$ is called *irreducible* if whenever $B = B_1 \cdot B_2$, then either $B_1$ or $B_2$ is the empty block. Let $\mathcal{I}(\mathcal{B}(G))$ represent the set of irreducible blocks in $\mathcal{B}(G)$. If $G$ is a finite Abelian group, then $\mathcal{B}(G)$ is a commutative atomic monoid (each nonempty block can be written as a product of irreducible blocks).

Given $G$ as above, a finite sequence $S = \{g_1, \ldots, g_k\}$ of not necessarily distinct elements of $G$ is called a *zero-sequence* if $\sum_{i=1}^k g_i = 0$. For simplicity, we write the zero-sequence $S$ multiplicatively as $\prod_{i=1}^k g_i^{n_i}$, where $n_i$ is the multiplicity of $g_i$ in $S$. $S$ is known as a *minimal zero-sequence* if it contains no proper subzero-sequence. It is easy to argue that the block $B = \prod_{g \in G} g^{v_g(B)}$ is irreducible in $\mathcal{B}(G)$ if and only if $\prod_{g \in G} g^{v_g(B)}$ is a minimal zero-sequence in $G$. The maximum value of $\sum_{i=1}^k n_i$ for a minimal zero-sequence $S$ is called the *Davenport Constant* of $G$ (see [9]) and denoted $D(G)$. While the value of Davenport's constant is known for a large class of finite abelian groups, no general formula is known to compute this value. If $S$ is a minimal zero-sequence of the form $\prod_{i=1}^k g_i^{n_i}$ and $G$ is a torsion group, then set $k(S) = \sum_{i=1}^k \frac{n_i}{|g_i|}$ where $\mid g_i \mid$ denotes the order of $g_i$ in $G$. $k(S)$ is called the *cross number of* $S$ (see [9]).

If $B$ is a block in $\mathcal{B}(G)$ and $B = B_1 \cdots B_t$, where each $B_i \in \mathcal{I}(\mathcal{B}(G))$, then $t$ is called a *length* of $F$. The set $\mathcal{L}(B) = \{t \mid t \text{ is a length of } B\}$ is commonly referred to as the *set of lengths of* $B$. If $G$ is a finite abelian group, then one can compute the following invariants: $L(B) = \max \mathcal{L}(B)$, $\ell(B) = \min \mathcal{L}(B)$, $\rho(B) = \frac{L(B)}{\ell(B)}$ and $\rho(\mathcal{B}(G)) = \sup\{\rho(B) \mid B \text{ is a block in } \mathcal{B}(G)\}$. $\rho(B)$ is known as the *elasticity* of $B$ and $\rho(\mathcal{B}(G))$ as the *elasticity* of $\mathcal{B}(G)$ (see [3] or [24]). It is well known that $\rho(\mathcal{B}(G)) = D(G)/2$ (see Proposition 3 in [4]) but not much is known about the set $\{\rho(B) \mid B \text{ is a block in } \mathcal{B}(G)\}$. Since there is a connection between the elasticity and the cross number (see [13]), and the set of cross numbers of the minimal zero-sequences of a $p$-group is known (see [11]), the following problem arises.

**Problem.** *Let $G$ be a $p$-group. Compute the set $\{\rho(B) \mid B$ is a block in $\mathcal{B}(G)\}$.*

The problem is clearly accessible to an advanced undergraduate mathematics student and would likely contain an interesting programming component.

**Stability and Chaos in Competitive Discrete Models** by Saber N. Elaydi

In this project, we study competitive discrete models. We start with the analysis of the growth dynamics of two discretely reproducing populations in competition. If $x_i(n)$ is the population density of species $i$ at generation $n$, $i = 1, 2$, then a reasonable model may be given by the nonlinear difference equation [17]

$$\left.\begin{aligned} x_1(n+1) &= x_1(n) g_1(x_1(n) + x_2(n)) \\ x_2(n+1) &= x_2(n) g_2(x_1(n) + x_2(n)) \end{aligned}\right\} \qquad (1)$$

where $g_i : [0, \infty) \to [0, \infty)$ is a strictly decreasing smooth function and $\lim_{x \to \infty} g_i(x) = 0$ for $i = 1$ and 2 [41, 31, 20]. The students are asked to address the following questions. **(i)** If $g_i(x_1 + x_2) = e^{r_i - k_i(x_1 + x_2)}$, $i = 1,\ 2$, find conditions under which species 2 ultimately excludes species 1. **(ii)** In general, find conditions under which species 2 ultimately excludes species 1.

A population may be subject to external agents (farmers, hunters, for example) which influence the growth dynamics. When an agent acts to increase a population, the action is called planting or stoking. A reasonable model for two competitive species with planting for species 1 is given by the difference equation [41]

$$\left.\begin{array}{l} x_1(n+1) = x_1(n)g_1(x_1(n) + x_2(n)) + p(x_1(n)) \\ x_2(n+1) = x_2(n)g_2(x_1(n) + x_2(n)) \end{array}\right\} \quad (2)$$

The students are asked to address the following. **(i)** Show that every orbit in (2) is bounded. **(ii)** If $g_i(x_1 + x_2) = e^{r_i - k_i(x_1+x_2)}, i = 1,\ 2$, find conditions under which species 2 ultimately excludes species 1. **(iii)** In general, find conditions under which species 2 ultimately excludes species 1. **(iv)** Constant Planting: If $p(x_1(n)) = \alpha, g_i(x_1 + x_2) = e^{r_i - k_i(x_1+x_2)}, i = 1, 2$, find conditions under which there is a stable coexistence between the two species. **(v)** Variable Planting: $p(x_1(n)) = \alpha * x_1(n)$. **(a)** Obtain conditions for the positive period 2 orbit to attract all positive solutions except for a one dimensional invariant manifold. **(b)** Obtain conditions under which both species in System (2) persist [19]. **(c)** Numerical simulations indicate that the period 2 orbit undergoes period-doubling bifurcation leading to chaos [18, 16]. Investigate analytically and graphically the bifurcation diagram, the existence of a Feigenbaum sequence and the Feigenbaum number. Finally, investigate when chaos occurs. **(d)** Let $g_i(x_1+x_2) = e^{r_i - k_i(x_1+x_2)}$, $i = 1,\ 2$. If $k_1 = k_2 = 1, \alpha = 0.5$ and $r_1 = 1.5$, show that if $1.5 + \ln 2 \leq r_2 \leq 3.411822071$, then System (2) has a positive period 2 orbit that attracts all positive solutions except for a one-dimensional invariant manifold.

**Dynamical Properties of Continuous Maps of the Interval** by Roberto Hasfura

We propose here that student participants research the dynamical properties of certain families of continuous functions of the interval $[0, 1]$. Many recent results showcasing surprisingly complex behavior of very simple-looking (for example, piecewise linear) maps, as well as others yet to be discovered, are accessible to students equipped with only the most elementary mathematical tools in the undergraduate curriculum. ([25] is an example.) Such research would in fact build on the successful work carried out by students who participated in REUs that took place at Trinity in the summers of 1997, 1998 and 1998. Collectively, those students have considered the dynamical properties (especially insofar as periodic points are concerned) of the families (i) $T_\omega(x) = \omega(1/2 - |x - 1/2|)$, $0 < \omega \leq 2$ (tent maps); (ii) $f_{m,e}(x) = mx$ or $me$ or $m(2 - x)$ depending on whether $0 \leq x < e$ or $e \leq x \leq 2 - e$ or $2 - e < x \leq 2$, where $m > 1$ and $me \in (2 - e, 2]$ (trapezoidal maps); and (iii) the piecewise-linear maps of $[0, 1]$ with three pieces, the 'middle one' mapping onto $[0, 1]$. Some results obtained during our 1999 Summer REU Program concerning a subfamily of the family in (iii) suggest some avenues of student research.

For specific examples, let's consider the two-parameter family of continuous maps $f_{a,b}$ of $[0, 1]$ given by

$$f_{a,b}(x) = \begin{cases} \frac{1-a}{b}x, & 0 \leq x \leq a \\ \frac{1}{1-2b}(x-1) + \frac{b}{1-2b}, & a < x \leq 1 - a \\ \frac{1-a}{b}(x-1) + 1, & 1 - a < x \leq 1 \end{cases}$$

where $0 \leq a \leq 1$ and $0 < b < 1/2$. A number of dynamical properties of this family were also established during the summer of 1999([36]), including some that beg for closer examination. The following surprising dichotomy provides one good such example: for $1/4 < b < 1/2$, there exist numbers $a_k$ increasing to $1 - b$ such that the map $f_{a,b}$ has a periodic orbit of length $2\cdot(2k+1)$ if and only if $a < a_k$; however, for $0 < b < 1/4$ and for all $0 < a < 1 - b$ the map $f_{a,b}$ always has periodic orbits of length six.

Another question considered in the summer of 1999 was that of possible topological conjugacies between maps in the family. A student constructed an interesting conjugacy between $f_{0,1/3}$ and $f_{0,2/5}$. The construction, fractal-like in its recursive nature, possibly gives rise to a conjugacy which appears to be almost everywhere non-differentiable. Elucidating this and other related questions (for instance: given $b$ and $a' \neq a''$ with no $a_k$ (as described above) satisfying $a' < a_k < a''$, are the maps $f_{a',b}$ and $f_{a'',b}$ topologically conjugate?) appears possible for motivated mathematics majors.

There are other questions that, albeit of slightly different flavor, are of relevance to the dynamical behavior of maps of the interval and that may be susceptible to the efforts of bright undergraduates. Students may be asked whether a given map of the interval admits an absolutely continuous (with respect to Lebesgue) invariant measure and, if so, what the statistical properties of that measure are. Finally, questions about the dynamical properties of families of maps that interact locally may be asked of the students. These questions are of much current interest [7]. Their answers can be glimpsed through computer experimentation and sometimes proved with mathematical tools available to undergraduates.

**Multiple Objective Linear Programming and Interior Point Algorithms** by Allen Holder

A multiple objective linear program (MOLP) is an optimization problem where one attempts to optimize several linear functions relative to linear constraints. The standard form of an MOLP is $\min\{Cx : Ax = b, x \geq 0\}$, where $C \in \mathbb{R}^{p\times n}$ and $A \in \mathbb{R}^{m\times n}$. Because there are multiple objective functions, to find an optimal solution really means to find a *pareto optimal solution*. This type of solution is a feasible vector, $x$, such that any other feasible vector that decreases one of the objective functions also increases another objective function. The set of pareto optimal solutions is called the *efficient frontier*. From a practical standpoint, it is useful to know how the objective functions behave on this entire set. Recent investigations into parameterizing the efficient frontier are found in [15].

Although interior point algorithms have revolutionized the field of linear programming (LP) [40], very little has been done with their connection to MOLP, exceptions being [1, 5]. Recently, Caron, Greenberg, and Holder [8] have developed new techniques to prove convergence of the analytic center path, which most interior point algorithms trace to optimality. This path terminates at the *analytic center solution* [33], which is a unique, interior solution. The uniqueness of this LP solution finally allowed those using LP to discuss a common solution that is readily computable. Hence, post solution analysis and interpretation could be based on a solution that is easily referenced. Using the analytic center solution, many mathematicians have worked on (re)inventing the topics of LP post solution analysis, see [22, 23, 28, 30, 29, 34]. The analytic center solution of an LP also induces the *optimal partition* [33], which completely defines the optimal face.

The new results of Caron, Holder, and Greenberg [8] have natural extensions to MOLP and may allow new classes of algorithms for this problem statement. Furthermore, there are several theoretical questions to explore, a few of which are now presented.

**Question 1.** *How does one define a unique, computable pareto optimal solution comparable to the analytic center solution for LP?*

Once the above definition is established, we are highly motivated to have a complete understanding of the information this solution contains. The following question demonstrates the type of information that is sought.

**Question 2.** *Is the analytic center pareto optimal solution somehow interior to the efficient frontier? Does it define an interior facet of the efficient frontier?*

Meaningful answers to these questions would be extremely interesting. Because there is so little known about the above topics, students will find it a fruitful area to work. Moreover, several modern software routines can be used to gain insight. Such computer work is often tangible and rewarding to beginning researchers.

**Jump System Analysis** by Vadim Ponomarenko

Jump systems are an exciting new combinatorial theory, with diverse applications throughout combinatorics. First described in 1995, jump systems simultaneously generalize (see [6]) two generalizations of matroids: delta-matroids and integral polymatroids. They naturally model bidirected graphs, and degree sequences of graphs [32]. They are also the lattice points in bisubmodular polyhedra [6].

Jump systems have an appealingly simple definition that lends itself well to undergraduate investigation. Fix $n > 0$, and let $J$ be a subset of $\mathbf{Z}^n$. Fix the $L^1$ metric, where for $x, y \in \mathbf{Z}^n$, $d(x, y) = \sum_{i=1}^{n} |x_i - y_i|$. Then $J$ is a jump system provided it satisfies the following axiom:

**(JS)** $\forall x, y \in J, \forall z_1$ with $x \xrightarrow{y} z_1$, then either: 1) $z_1 \in J$, or 2) $\exists z_2 \in J$ with $x \xrightarrow{y} z_1 \xrightarrow{y} z_2$,

where $x \xrightarrow{y} z$ means $d(x,z) = 1$ and $d(x,y) > d(z,y)$. Students first will consider low-dimensional jump systems. While one-dimensional jump systems are easily understood (they have no gaps of size larger than one), a taxonomy of two-dimensional jump systems remains open.

Currently there is only a limited collection of jump system results (See [6], [32], [35]). For example, the cartesian product of jump systems is a jump system, the intersection of a jump system with a box is a jump system, and the projection of a jump system onto a lower-dimensional subspace is a jump system.

Students will be asked to consider these and similar, somewhat more complex, questions. They will be expected to construct their own arguments for proof; some of the existing proofs are cumbersome and might be improved upon. Due to the young nature of the field, there is also great potential for discovery of other properties of similar flavor.

Next, students will be asked to consider the primary open question of jump system theory: the intersection problem. The intersection problem is whether two given (abstractly defined) jump systems have a point in common. There are several partial solutions ([32], [35]), but the main question remains open. Students will be asked to consider special cases. For example, the cases where the elements of the jump system have entries whose sum is zero, or more generally a constant, or more generally a class of constants (such as the even integers).

Students will also be asked to consider algorithmic and computational aspects of solving the intersection problem. If there is no good solution, students will consider whether there is at least a (somewhat) efficient search that improves upon naively checking each element of one jump system against each element of the other.

**Finite Upper Half-Planes and Sums of Squares** by Holly Rosson

Given a positive integer $n$, in how many ways can we write $n$ as the sum of $k$ squares? Geometric, algebraic, and analytic methods have been employed to answer this question. One of the most elegant tools used in tackling this problem has been the theory of automorphic forms. Loosely speaking, an automorphic form is a meromorphic function on the complex upper half-plane that transforms nicely under the action by linear fractional transformations of $SL_2(\mathbf{Z})$. As an example, the theta function

$$\theta(z) = \sum_{(x_1,x_2,\ldots,x_k)\in\mathbf{Z}^k} e^{2\pi i(\sum_i x_i^2)z} \qquad (z \in \mathbf{C})$$

is an automorphic form whose $m^{th}$ Fourier coefficient is equal to the number of ways of writing $m$ as the sum of $k$ squares. By writing the theta function as an explicit linear combination of other automorphic forms called Eisenstein series whose Fourier coefficients are known, we obtain formulas for the number of ways of writing $n$ as the sum of $k$ squares for various $k$.

This work has been mimicked in other settings. For instance, in [26], the authors study a theta function on an analog of the classical upper half-plane to find the number of ways of writing a polynomial over a finite field as the sum of squares of $k$ polynomials (whose degrees are bounded by a fixed integer). Further, a finite analog of an upper half-plane has been the object of considerable study in the past few years (see [37]). This space has many similarities to that given in [26], the most notable being that they are both discrete spaces. This makes computations involving this "finite upper half-plane" significantly easier than the classical computations. Research in this area is accessible to undergraduates; the required background knowledge is limited to understanding $\mathbf{Z}/p\mathbf{Z}$.

In this project, the students will investigate how this upper half-plane can be used to answer questions such as finding the number of ways of writing an element in a finite field as the sum of $k$ squares (and more generally, computing the representation numbers of other quadratic forms over a finite field). While some of these questions have already been answered using various elementary techniques, they have not been answered using "automorphic forms" on this upper half-plane. It will be a good project in that the students may not only develop an entirely different solution technique, but it will also introduce them to automorphic forms and number theory, an area that has been studied heavily in the past and continues today.

## 3 The Research Environment

Trinity University is a leading undergraduate institution. In 2000, *U.S. News and World Report* ranked Trinity as the "number one Western Regional University" for the 9th consecutive year[4]. Also in 1999, *The Insider's Guide to the Colleges* described Trinity as follows:

> "With its large endowment, Trinity has attracted top rate professors to teach small classes. As a result, Trinity continually receives high rankings...as one of the top small liberal-arts universities west of the Mississippi."

Trinity routinely attracts about 30 national merit finalists per year, and the average SAT and ACT scores for entering students vary slightly from 1270 and 27, respectively. Upon graduation about 35% of Trinity's mathematics majors go to graduate school, another 25% become teachers, and the remaining graduates seek jobs in industry (often in actuarial science and software engineering). According to a recent study[5], between 1920 and 1995 13 PhDs were granted in mathematics to students whose bachelors degrees were from Trinity. This ranks Trinity 32nd out of 253 institutions which do not grant doctoral degrees (i.e., institutions with Carnegie Classifications MAI or MAII). From 1986 through 1995, this same report shows 2 PhDs granted in mathematics to students with bachelors degrees from Trinity (ranking Trinity 31st of 253). We feel that our Mathematics graduates have a proven record of success.

Undergraduate research is heavily emphasized within the division of Science, Mathematics, and Engineering. The departments of Chemistry, Computer Science, and Mathematics have each had REU programs for the last several summers. Each of these programs has been a success, with both faculty and students enjoying fruitful research projects. In the past, the participants in the Mathematics and Computer Science REU programs have been encouraged, through the use of an afternoon tea-hour, to socialize and exchange ideas. In the future we hope to expand this group to include the numerous undergraduate research students from Chemistry, Physics and Biology on our campus during the summer months. Students find that these informal meetings provide a time to discuss a variety of scientific topics.

To emphasize the quality of the research environment here at Trinity, we will: 1) discuss how undergraduate research is implemented as part of the mathematics degree at Trinity, 2) highlight the principal investigators qualifications and prior experiences with undergraduate research, and 3) show a history of past support. The discussion in the remainder of this Section not only supports the contention that the Trinity Mathematics Department is one of the strongest academic departments at the university, but also reinforces the claim that the Principal Investigators of this proposal are adept at working on research projects with undergraduates.

A student receiving an undergraduate mathematics degree from Trinity University must complete a Senior Research Project. This requirement is incorporated in the Majors' Seminar (Math 3194 and Math 4194), a mandated part of both the junior and senior curriculum. The Senior Project is a year long activity which begins in the fall of the student's Senior year with the selection of a Faculty Project Advisor and Project Topic. The student further selects a three member faculty committee (including the Project Advisor) to evaluate the project and presents both an oral and written Project Proposal during the fall semester of their Senior year. The final Senior Project (completed by the end of the spring semester) consists of both a written and oral part. The written document must contain substantial mathematical content, and during the last month of spring classes, the student presents a 20 to 25 minute oral summary of their findings. In an effort to make sure that each student has a successful project, individual students meet regularly (ideally once a week) with their Project Advisors. The Department is proud of the progress of this program which was first instituted as a graduation requirement in 1992. Examples of the written portion of two recent Senior Projects can be found at [39], and a document outlining the project guidelines is at [38].

The faculty's activities described above are not the full extent of their involvement in undergraduate research. We now highlight this further activity, as well as various distinctions of the principal investigators. The Program Director, Dr. Scott Chapman, has been a faculty member in Trinity's Mathematics Department

[4] http://www.usnews.com/usnews/edu/college/rankings/wstunivs/wstu_a2.htm

[5] *Baccalaureate Origins of Doctoral Recipients*, 8th edition, Franklin & Marshall College, 1998.

for 13 years. Among his list of over 40 articles authored or co-authored in refereed mathematical publications such as *Proceedings of the American Mathematical Society*, *Israel Journal of Mathematics*, *Journal of Algebra*, *Journal of Number Theory* and *Journal of Pure and Applied Algebra*, are 4 papers co-authored with undergraduate students. In 1994 he received a Senior Fulbright Research Scholarship and spent part of the spring 1995 semester at Karl-Franzens-Universität in Graz, Austria, studying integral domains and monoids. He has also received fellowships for foreign study in Italy from the Consiglio Nazionale delle Ricerche (1994), and in Germany from the Deutscher Akademischer Austauschdienst (1996). Dr. Saber Elaydi serves as Co-Editor in Chief of the *Journal of Difference Equations and Applications* and has authored or co-authored over 60 research articles on topics in difference equations, topological dynamics, and differential equations. These articles have appeared in journals such as *Proceedings of the American Mathematical Society*, *Applied Mathematics and Computation*, *Journal of Mathematical Analysis and Applications*, *Nonlinear Analysis*, and *Funkcialaj Ekvacioj*. In addition, he is the author of two recent textbooks aimed at an undergraduate audience: *An Introduction to Difference Equations* (Springer-Verlag, 1996 and 1999), and *Discrete Chaos* (Chapman and Hall, 2000). He is spending the fall 2000 semester as a Senior Fulbright Research Scholar on the West Bank. In addition to mentoring REU students, Dr. Elaydi has supervised the research of five Trinity students who were funded by his RUI-NSF grant (1997-2000). Dr. Roberto Hasfura was one of the original investigators on Trinity's first funded Mathematics REU Program, and mentored REU students in each of the grant's three years. His areas of expertise are in ergodic theory and dynamical systems, and he has published papers in several prestigious journals, including the *Transactions of the American Mathematical Society*. He has co-authored a paper with one REU student which is currently under review. Professor Hasfura is serving in his second year of a three year term as Department Chair. Over the last 5 years, Professors Chapman, Elaydi and Hasfura have published over 52 papers in refereed mathematical publications. Since the selection of an REU topic can be a critical factor in the success of a project, the expertise demonstrated by these investigators is invaluable to the success of this program.

While these three principal investigators clearly demonstrate their expertise in their respective areas, the remaining Co-Principal Investigators offer equally as impressive credentials. The Department has been fortunate to make three tenure-track appointments over the past two years, and these appointments comprise the remainder of our proposed REU faculty. Dr. Allen Holder holds the Ph.D. degree from the University of Colorado at Denver and specializes in Mathematical Programming. Dr. Holder is already the author or co-author of three refereed publications. In 1993, he directed undergraduate and high school research projects supported by NASA at The University of Southern Mississippi. During his first year at Trinity, Dr. Holder acted as Project Advisor for two Senior Research Projects and has already won two university level teaching awards. His dissertation has been nominated for the National Council of Graduate Schools Award and his paper [27] has been nominated for the William Pierskalla Award for best paper in Health Applications. Dr. Vadim Ponomarenko joined the department last year after completing the Ph.D. degree in Mathematics at the University of Wisconsin at Madison. In addition, Dr. Ponomarenko holds a Master's degree in Computer Science from that same institution. Professor Ponomarenko brings expertise to the grant in the exciting new area of Combinatorics known as "Jump System Analysis." His enthusiasm for Combinatorics has already attracted the largest undergraduate enrollment to this course at Trinity since the early 1990's. Joining the department this fall is Dr. Holly Rosson. Dr. Rosson completed the Ph.D. degree this June at Dartmouth with a dissertation in Number Theory, and already has a paper based on her doctoral work under review at a refereed journal. She has 8 years of experience in university level teaching as a Graduate Teaching Fellow at both Dartmouth and the University of Vermont. To summarize, all six of this proposal's principal investigators have the experience required to guide intelligent, young minds through their initial exposure to mathematical research.

The Mathematics Department at Trinity has a decade long history of success with grant applications. Aside from the already mentioned successful REU and RUI proposals in 1997, over the last eleven years, the mathematics department has received 3 PEW Foundation grants to support undergraduate research and to develop courses. Over this same time period, 2 additional NSF grants were awarded to the mathematics department. These were awarded in 1989 and 1994 and were used to set up computer labs. These computer labs are equipped with several software packages and will be available for the REU activities proposed.

# 4 Student Recruitment and Selection

While we plan to recruit students nationwide, an emphasis will be placed on recruiting students belonging to under represented minority groups in South Texas. This is an endeavor we clearly fell short of in our proposed goals for the 1997-1999 program (see Section 6, *Results from Prior REU-NSF Support*), and we are prepared to devote added attention to this area so that future efforts in this direction are successful. It is easy for us to attribute the lack of minority participation in our earlier program to the number of minority applications we received. Our plan to invite students from predominately Hispanic local colleges and universities to our seminars and colloquiums generated almost no interest in the program. Our approach to recruiting under-represented minorities was evidently not aggressive enough.

There are eight 4 year institutions in South Texas which have high Hispanic enrollments and very little capacity for research in mathematics and related fields. We list these institutions, with their percent of undergraduate Hispanic enrollment[6], in the following table.

| Institution and Location | Undergrad. Hispanic Enroll. | Institution and Location | Undergrad. Hispanic Enroll. |
|---|---|---|---|
| Univ. of Texas at San Antonio | 40% | Texas A&M Kingsville | 62% |
| St. Mary's Univ. (San Antonio) | 65% | Texas A&M Corpus Christi | 37% |
| Incarnate Word (San Antonio) | 51% | St. Edward's (Austin) | 25% |
| Our Lady of the Lake (San Antonio) | 62% | Texas Pan American (Edinburg) | 85% |

Our present plan is to target these schools for applications from qualified Hispanic students. Since our previous attempts to get such students to visit the Trinity campus were unsuccessful, we will travel to them. During the early period of February (see the timetable in Section 1, *Introduction and Overview*), we will send one or more of this grant's principal investigators to visit each of the campuses listed above. If possible, investigators will schedule a colloquium at these schools, as well as arrange individual interviews with students who express interest in the program. These contacts will be followed up during the first week of March by the Program Director with telephone calls and electronic mail messages. When possible, we will have these potential applicants visit our Department to get a first hand look at our program. The success of the Trinity Admissions Department with campus visits for potential students convinces us that getting potential applicants here for a visit is extremely important. Having the continued involvement of Dr. Roberto Hasfura, who is of Hispanic origin, in our program strengthens and makes our goal of increased Hispanic participation more achievable.

This attempt to emphasize Hispanic participation is based on the demographics of the region in which we are located and is not meant to imply that we will not pursue other under represented minority groups or women. Our past program did attract a much higher percentage of female students than minority students (exact participation figures are supplied in Section 6), and we hope that the efforts outlined above will also help us to increase the number of applications from these groups as well. Moreover, the Trinity Mathematics Faculty has had great success in attracting female students, with more than 50% of our present mathematics majors being women. We will consider a minority and/or female participation in our program of lower than 60% to be a failure.

We plan to slightly alter our earlier general approach to recruitment. While it is still practical to distribute materials to schools in our area by regular mail, our contacts outside of Texas will be handled almost exclusively through the world wide web and electronic mail. During the last two years of our previous program, we found that almost all the applications submitted from outside of Texas were downloaded from our web site. With this new emphasis in the process of distributing applications, we plan to expand our world wide web presence. Our web page will now not only contain links to download the application and reference forms, but also information from the individual investigators on possible project topics as well as a summary of the success of our past program.

[6] Information in the table is the latest available from each institution's Office of Institutional Research

We have found our application materials to be more than adequate and plan little change in the formal process of applying. Aside from asking for basic information, the application asks students to list all their formal coursework in mathematics, including the texts used in their courses and the chapters covered from the texts. We ask the applicants to attach a one page statement describing their general mathematical background and reasons for their interest in our program. A complete application also includes an official transcript and two faculty reference letters. We will rank student applications for the REU program according to the following criteria: 1) scholastic achievement, 2) interest in mathematics, and 3) motivation. The selection of candidates will be done by the six Principal Investigators on a ten point scale. Up to six points are awarded to applicants based on scholastic achievement (supported by the transcript and faculty references). The other two criteria are awarded up to two points each, and are assessed from the written statement supplied by the student.

# 5 Evaluation

During Trinity's 1997-1999 REU Program, our evaluation procedures consisted of a written form given to the participants in the last week of the program. The form contained two open ended questions, and some quotations from student responses on these forms can be found in Section 6 of this proposal. While we found this form useful, we believe that to significantly improve the program, the evaluation procedures must be broadened and brought more into line with the objectives of the program as stated in Section 1. Hence, we propose a yearly three step evaluation process which consists of: 1) an expanded written Exit Evaluation Form completed anonymously by participants during the last week of the program, 2) Exit Interviews where the students are encouraged to openly discuss their experience with the Program Director and express ideas which they feel the written form did not address, and 3) a yearly written follow up with each past Trinity REU participant to gauge how their REU experience is contributing to their current career plans. The majority of the questions on the Exit Evaluation Form will contain a measured scaled response. The results of these procedures will be reviewed by the principal investigators the week following the end of the summer program and will be included in our yearly progress report to the NSF. Below we outline the questions which will make up both written evaluation forms and discuss the structure of the Exit Interview.

The proposed Exit Evaluation form consists of 10 questions. For the first 7 questions, the students are asked to answer on a 6 point scale (0=strongly disagree, 1=disagree, 2=slightly disagree, 3=no opinion, 4=slightly agree, 5=agree, 6=strongly agree). The questions are as follows.

1. The Trinity REU Program offered me the opportunity to better understand the nature of mathematical research.
2. The facilities on the Trinity campus enhanced my experience.
3. I found the Topic Colloquiums during the first week of the program useful and interesting.
4. My project challenged me mathematically.
5. My Project Director was attentive to my questions and ideas.
6. I received sufficient feedback on my work throughout the project.
7. My REU experience has influenced my future career plans.

We also propose the following 3 free response questions.

8. Comment on what you feel are the strongest and weakest aspects of the Trinity REU Program.
9. Comment on what you feel we can do to improve the program in the coming years.
10. If appropriate, comment on the use of technology in your project.

We set as a goal an average response from the participants of at least 5.0 on each question numbered 1–7. Should the average on such a question fall below this level, a written plan addressing corrective measures will be included in our following yearly progress report to NSF. We will include a similar plan should concerns expressed in questions 8–10 be aired by more than one student.

We feel that the Exit Interview is an opportunity to obtain feedback from the students which otherwise might be difficult to discern from a written format. Possible questions during this process might include:

i. Are you more or less likely to now attend graduate school in a mathematical science? In addition, how likely are you to continue work in the mathematical field you have studied in the REU Program?

ii. How rewarding was your mathematical experience here? Moreover, how rewarding was your entire experience in the Trinity REU Program?

iii. Would you encourage a friend to apply to this program? If the answer is no, then what can we do to make the program more appealing?

Finally, the Exit Interview gives us the chance to check the student's permanent address and convince them of the importance of participating in our yearly follow up surveys.

A yearly "Alumni Survey" may be a better gauge of the true success of our program. Besides offering us the opportunity to check on the career status of our past students, it allows us to ask several of the questions above in retrospect. Of particular interest to the investigators are written responses to questions similar to questions 7, 8, 9, ii and iii. We currently plan to distribute this questionnaire early in March to allow the past participants ample time to respond before our yearly assessment meetings in late July. Whenever possible, the Alumni Survey will be distributed electronically. Negative responses received from more than one student on any particular question in either the Exit Interviews or the Alumni Surveys will be addressed in writing in our annual report to NSF.

## 6 Results from Prior REU-NSF Support

**Project Director**: *Saber N. Elaydi* **Title**: Undergraduate Research Experience in Mathematics
**Award**: DMS-9619837 **Award amount:** $72,512
**Award period**: 5/15/97 - 4/30/2000

During each of the three summers of our program, we received approximately 50 applications for admission. We were exceedingly fortunate in the selection of our REU students. They were very bright and self-motivated. The students worked extremely hard and made many mathematical discoveries. All of the supervisors were very impressed by the insights and deep understandings the students gained in a short period of time. The problems given to the students were by no means trivial (several were known open problems in the mathematics literature). As an outcome of the students research, two papers written jointly with supervisors have been accepted and or published in refereed mathematical journals and a third paper is currently under review.

The student's level of satisfaction with the program was consistently high over the program's three summers. One student commented, "My first time to do research one-on-one with a professor was a great experience." A second student said, "I thought that the flexibility of the faculty to let the students explore options within the research project was also exemplary." A third student wished that the program had been longer and in general the students liked the daily tea-hour. Other students responded in the following manner in their evaluations of the program:

> "I have really enjoyed my summer research experience at Trinity. I feel that it has definitely given me a better understanding of what mathematics research is like. This will be very helpful in making decisions about graduate school."

"The two main strengths of Trinity's REU are the high expectations held for its participants and the small size of the program. The research in which we have participated, though basic enough to be accessible to undergraduates, has also been ambitious enough to make us stretch and enhance our mathematical knowledge."

While we are all pleased with the scientific success of the program, we did not succeed in achieving one of our goals. An important aim of Trinity's REU was to attract a sizable number of minority students. The program fared much better in attracting females. During the summers of 1997 and 1998, 40% of our students were female, and during the summer of 1999 16% were female. We tried very hard to recruit Hispanic students but, unfortunately, fell far short of our goal.

Three of the projects undertaken by our REU students led to significant results which have either been published, are pending publication, or are still under review by a refereed mathematical journal. Following a table which lists all the students who participated in our program, their year of participation, their home institution, their project advisor[7], and their project topic, we give a brief description of these three projects.

**Student Participants in the 1997-1999 Trinity Mathematics REU Program**

| Student | Year | Home Institution | Advisor | Topic |
|---|---|---|---|---|
| Noelle Dexheimer | 1997 | Texas A&M University | Saphire | Linkage Analysis |
| Kala Schrager | 1997 | University of Oregon | Chapman | Zero-Sequences |
| John May<br>Aaron Heap<br>Phillip Lynch | 1997<br>1997<br>1997 | University of Oregon<br>Texas Christian University<br>Washington University | Elaydi<br>&<br>Hasfura | Dynamics<br>of<br>Tent Maps |
| Becky Cantonwine<br>Jason Heller | 1998<br>1998 | Hanover College<br>Elizabethtown College | Hasfura | periodic orbits of<br>trapezoidal maps |
| Michael McQuistan | 1998 | University of Nebraska | Bailey | Generalizations of the<br>Mean Value Theorem |
| Jeremy Herr<br>Natalie Rooney | 1998<br>1998 | University of Oklahoma<br>University of Texas at Austin | Chapman | Diophantine<br>Monoids |
| Bryant Mathews<br>Nick Neumann | 1999<br>1999 | Harvard University<br>Texas A & M University | Elaydi | Behavior of Discrete<br>Species Models |
| Steven Steinke<br>Matthew Westerhoff | 1999<br>1999 | University of Arizona<br>Univ. of Texas at San Antonio | Hasfura | Dynamics of Certain<br>Piecewise Maps |
| Vic DeLorenzo<br>Holly Swisher | 1999<br>1999 | Grove City College<br>University of Oregon | Chapman | Factorization in<br>Block Monoids |

**Project I.** During the summer of 1997, *John May*, *Aaron Heap* and *Phillip Lynch* worked with *Drs. Elaydi* and *Hasfura*. They investigated the dynamical behavior of the family of tent maps $T_w(x) = w(0.5 - |x - 0.5|)$, for $w > 0$. The students determined completely the nature of the sequence $\{w_k\}$, where $w_k$ is the infimum of the values of $w$ for which the corresponding map has points of prime period $k$. Another result is the discovery of the existence of transitive attractors for some of these one-dimensional maps. Most of the results were first conjectured on the basis of vast empirical evidence obtained using Maple. A joint paper co-authored by *Dr. Hasfura* and *Mr. Lynch* involving this work is pending publication in a refereed mathematical journal.

**Project II.** During the summer of 1998, *Jeremy Herr* and *Natalie Rooney* worked under the supervision of *Dr. Chapman* on problems involving factorizations of elements into irreducible elements in monoids. Let $\mathbb{Z}$ be the set of integers and $\mathbb{N}$ the set of nonnegative integers. The basis of the project was to study the behavior of factorizations of elements into sums of irreducible elements in additive monoids of the type $M(a_1, \dots, a_n) = \left\{(x_1, \dots, x_n) \in \mathbb{N}^n : \sum_{j=1}^{n} a_j x_j = 0\right\}$, where $a_1, \dots, a_n \in \mathbb{Z}$. The students successfully found the number of nonassociated irreducible factorizations of $x \in M(a_1, \dots, a_n)$, for all $x \in M$, for

[7]Professor Donald Bailey joined the program during the summer of 1998 in place of Professors Elaydi and Saphire

the monoids $M(1,1,-1,-1)$, $M(1,1,-2)$, $M(1,1,1,-2,)$, and $M(1,1,1,\ldots,1,-2)$. Moreover, they found a general formula for counting the number of nonassociated factorizations of an element into irreducible elements in an algebraic number ring of class number 2. A joint paper written by *Mr. Herr, Ms. Rooney* and *Dr. Chapman* containing these results has been published by the *Journal of Number Theory* [12].

**Project III.** During the summer of 1999, *Vic DeLorenzo* and *Holly Swisher* worked under the direction of *Dr. Chapman.* Their work centered on the asymptotic behavior of irreducible factorizations in block monoids. Let $G$ be a finite abelian group written additively and $\mathcal{B}(G) = \{B \in \mathcal{F}(G) \mid \sum_{g\in G} v_g(B)g = 0\}$ denote the *block monoid* considered earlier in Section 2. As mentioned previously, $\mathcal{B}(G)$ is an atomic monoid. If $x$ is a block in $\mathcal{B}(G)$, then set $\eta(x)$ to be the number of nonassociated irreducible factorizations of $x \in R$, and for a positive integer $d$, $\bar{\eta}_d(x) = \lim_{k\to\infty} \frac{\eta(x^k)}{k^d}$. In addition, set $\sigma(x) = \min\{d \mid d \in \mathbb{N}_0 \text{ and } 0 < \bar{\eta}_d(x) < \infty\}$. Mr. DeLorenzo and Ms. Swisher were able to prove several results concerning the function $\sigma(x)$. A paper centering on these results, written jointly with *Dr. Chapman*, has been accepted for publication by the *Semigroup Forum* [10]. Among the student's findings was the following: if $G$ is a finite Abelian group, then the following statements are equivalent: 1) $G$ is an elementary $p$-group for some prime integer $p$, 2) if $x = g_1^{n_1}\cdots g_t^{n_t}$ is an irreducible block in $\mathcal{B}(G)$ with $\sigma(x) = 1$, then $n_1 = n_2 = \cdots = n_t = 1$, 3) There exists a prime integer $p$ such that for all irreducible $x \in \mathcal{B}(G)$ with $\sigma(x) = 1$, $\bar{\eta}_1(x) = 1/p$, 4) There exists a positive rational $q$ such that for all irreducible $x \in \mathcal{B}(G)$ with $\sigma(x) = 1$, $\bar{\eta}_1(x) = q$.

**Current Activities of Past Participants:** A total of 16 undergraduate students participated in our REU program during the summers of 1997, 1998 and 1999. Three of these students (Steinke, Mathews and Neumann) are currently Seniors at their respective institutions and plan to graduate with undergraduate degrees in 2001 or 2002. All three are still majoring in a Mathematical Science and considering graduate study in a related field. Four other students have completed undergraduate degrees and gone on to private employment. Kala Schrager is a teacher at the Briarwood School for Learning Disabled Students in Houston, Texas and Becky Cantonwine is a secondary school teacher in Indiana. Natalie Rooney is a Software Engineer for Infoglide, Inc. in Austin, Texas and Matthew Westerhof has accepted a position with Applied Research Laboratories also in Austin. The remaining 9 students have gone on to graduate study in Mathematics or Statistics. Noelle Dexheimer completed a Master's degree in Statistics in August 2000 at Texas A& M University. The remaining 8 students have pursued graduate degrees in a mathematical science and we list their current institutions in the following chart. Notice that 7 of these 8 students have enrolled in *Group I* graduate institutions as defined by the *American Mathematical Society*[8].

| Student | Graduate School | Student | Graduate School |
|---|---|---|---|
| Aaron Heap | Rice University | Vic DeLorenzo | University of Illinois at Urbana |
| John May | Georgia Tech | Michael McQuistan | University of Wisconsin at Madison |
| Jeremy Heller | SUNY Stony Brook | Jeremy Herr | University of Michigan |
| Philip Lynch | University of Chicago | Holly Swisher | University of Wisconsin at Madison |

## Publications Resulting From the Grant

a) R. Hasfura and P. Lynch, Periodic points of the tent family, *submitted*, `http://www.trinity.edu/jhasfura/Reu`.

b) S.T. Chapman, V. DeLorenzo and H. Swisher, On The Asymptotic Behavior of Irreducibles in Block Semigroups, to appear in *Semigroup Forum*, `http://www.trinity.edu/schapman/finalsgf.tex`.

c) S.T. Chapman J. Herr and N. Rooney, A factorization formula for class number two, *J. Number Theory* **79**, 58–66, 1999.

[8]See *Notices Amer. Math. Soc.* **35**, pp. 532–533, 1988.

## References


[1] S. S. Abhyankar, T. L. Morin, and T. Trafalis Efficient faces of polytopes: Interior point algorithms, parameterization of algebraic varieties and multiple objective optimization, in Mathematical Developments Arising From Linear Programming, editors J. Lagarias and M. Todd, American Mathematical Society, pp. 319-341, 1990.

[2] I. Adler and R. Monteiro, A geometric view of parametric linear programming, *Algorithmica* **8**, pp. 161-176, 1992.

[3] D.F. Anderson, Elasticity of factorizations in integral domains: a survey, Lecture Notes in Pure and Applied Mathematics, Marcel Dekker **189**, pp. 1–30, 1997.

[4] D.F. Anderson and S.T. Chapman, On the elasticities of Krull domains with finite cyclic divisor class group, *Comm. Algebra* **28**, pp. 2543–2553, 2000.

[5] A. Arbel, A multiobjective interior primal-dual linear programming algorithm, *Computers & Operations Research* **21**, pp. 433-445, 1994.

[6] A. Bouchet and W. Cunningham, Delta-matroids, Jump Systems, and Bisubmodular Polyhedra, *Siam J. Disc. Math* **8**, pp. 17–32, 1995.

[7] L.A. Bunimovich and S. Venkatagiri, Onset of chaos in coupled map lattices via the peak-crossing bifurcation, *Nonlineariy*, **9**, 1281–1298, 1996.

[8] R.J. Caron, H.J. Greenberg, and A.G. Holder, Analytic Centers and Repelling Inequalities, Technical Report CCM#142, Center for Computational Mathematics, University of Colorado at Denver, Denver, CO, USA, 1999.

[9] S.T. Chapman, On the Davenport constant, the cross number, and their application to factorization theory, *Lecture Notes in Pure and Applied Mathematics*, Marcel Dekker, **171**, pp. 167–190, 1995.

[10] S.T. Chapman, V. DeLorenzo and H. Swisher, On The Asymptotic Behavior of Irreducibles in Block Semigroups, to appear in *Semigroup Forum*.

[11] S.T. Chapman and A. Geroldinger, On cross numbers of minimal zero sequences, *Australasian J. Comb.* **14**, pp. 85–92, 1996.

[12] S.T. Chapman J. Herr and N. Rooney, A factorization formula for class number two, *J. Number Theory* **79**, 58–66, 1999.

[13] S.T. Chapman and W.W. Smith, An analysis using the Zaks-Skula constant of element factorizations in Dedekind domains, *J. Algebra* **159**, pp. 176–190, 1993.

[14] P. Collet and J.P. Eckmann, Iterated maps on the interval as dynamical systems, *Birkhauser*, Boston, 1980.

[15] I. Das and J. Dennis, Normal-Boundary Intersection: A new method for generating Pareto Optimal point in Multicriteria Optimization Problems, *SIAM Journal on Optimization* **8**, pp. 631-657, 1998.

[16] R.L. Devaney, An Introduction to Chaotic Dynamical Systems, Benjamin/Cummings, Menlo Park, CA, 1986.

[17] S.N. Elaydi, An Introduction to Difference Equations, Second Edition, Springer-Verlag, New York, 2nd ed., 1999.

[18] S.N. Elaydi, Discrete Chaos, Chapman Hall/CRC, 1999.

[19] A. Fonda, Uniformly persistent semi-dynamical systems, Proc. Amer. Math. Soc. **104**, pp. 111-116, 1988.

[20] J.E. Franke and A.-A. Yakubu, Mutual exclusion versus coexistence for discrete competitive systems, J. Math. Biol. **30**, pp. 161–168, 1991.

[21] A. Geroldinger and F. Halter-Koch, Non-unique factorizations in block semigroups and arithmetical applications, *Math. Slovaca* **42**, pp. 641–661, 1992.

[22] H. Greenberg, The use of the optimal partition in a linear programming solution for postoptimal analysis, *Operations Research Letters* **15**, pp. 179-185, 1994.

[23] H. Greenberg, Rim Sensitivity Analysis from an Interior Solution, *SIAM Journal on Optimization* **10**, pp. 427-442, 2000.

[24] F. Halter-Koch, Elasticity of factorizations in atomic monoids and integral domains, *J. Théorie des Nombres Bordeaux* **7**, pp. 367–385, 1995.

[25] R. Hasfura and P. Lynch, Periodic points of the tent family, *submitted.*

[26] J. Hoffstein, K.D. Merrill, and L.H. Walling,, Automorphic forms and sums of squares over function fields, *J. Number Theory* **79**, pp. 301–329, 1999.

[27] A. Holder, Designing Radiotherapy Plans with Elastic Constraints and Interior Point Methods, Trinity University Mathematics, Report #49, 2000, in review.

[28] A. Holder, J. Sturm, and S. Zhang, The analytic central path, sensitivity analysis, and parametric programming, Technical Report CCM#118, Center for Computational Mathematics, University of Colorado at Denver, Denver, CO, USA, 1997.

[29] B. Jansen, J.J. de Jong, C. Roos, and T. Terlaky, Sensitivity analysis in linear programming: Just be careful!, *European Journal of Operations Research* **101**, pp. 15-28, 1997.

[30] B. Jansen, C. Roos, and T. Terlaky, An interior point approach to post optimal and parametric analysis in linear programming, Technical Report #92-21, Delft University of Technology, Delft, Netherlands, 1992.

[31] V.L. Kocic and G. Ladas, Global behavior of nonlinear difference equations of higher order with applications, Kluwer Academic Publishers, Dordrecht (1993).

[32] L. Lovász, The Membership Problem in Jump Systems, *J. Combin. Theory Ser. B* **1**, pp. 45–66, 1997.

[33] L. McLinden, An analogue of Moreau's proximation theorem, with applications to the nonlinear complementary problem, *Pacific Journal of Mathematics* **88**, pp. 101-161, 1980.

[34] R. Monteiro and S. Mehrotra, A General Parametric Analysis Approach and its Implication to Sensitivity Analysis in Interior Point Methods, *Mathematical Programming* **72**, pp. 65-82, 1996.

[35] V. Ponomarenko, Some Results on Jump Systems and Rota's Conjecture, *PhD Thesis, University of Wisconsin - Madison*, 1999.

[36] S. Steinke, The dynamical properties of a family of piecewise linear maps of the interval, *in preparation.*

[37] A. Terras, Eigenvalue problems related to finite analogues of upper half planes, *Forty more years of ramifications: spectral asymptotics and its applications* Texas A & M Univ., College Station, TX, pp. 237–263, 1991.

[38] Trinity University Mathematics Department Senior Project Guidelines, http://www.trinity.edu/schapman/majsem00-01.pdf.

[39] Trinity University Mathematics Department Student Technical Reports, http://www.trinity.edu/departments/mathematics/studtechreport.html.

[40] M. Wright, The Interior-Point Revolution in Constrained Optimization, *High-Preformance Algorithms and Software in Nonlinear Optimization*, R. Deleone, A. Murli, P. Pardalos, and G. Toraldo eds., pp. 359-381, Kluwer Academic Publishers, 1998.

[41] A. Yakubu, The effects of planting and harvesting on endangered species in discrete competition systems, Math. Biosc. **126**, pp. 1–20, 1995.

**Scott Thomas Chapman**

Biographical Sketch

Trinity University
Department of Mathematics
715 Stadium Drive
San Antonio, Texas 78212-7200
e-mail: schapman@trinity.edu

**Education**:

Ph.D. in Mathematics, The University of North Texas, Denton, Texas, 1987.

M.S. in Mathematics, The University of North Carolina at Chapel Hill, Chapel Hill, North Carolina, 1984.

B.S. (cum laude with Honors in Mathematics) in Mathematics and Political Science, Wake Forest University, Winston-Salem, North Carolina, 1981.

**Positions Held**:

Professor, Trinity University, 1999- .

Associate Professor, Trinity University, 1993-1999.

Assistant Professor, Trinity University, 1987-1993.

Teaching Fellow, The University of North Texas, 1984-1987.

Graduate Teaching Assistant, The University of North Carolina at Chapel Hill, 1981-1984.

Teaching Assistant, Wake Forest University, Fall 1980.

**Synergistic Activities**:

J. William Fulbright Research Scholar to Austria, 1995.

Edited (with S. Glaz) *Non-Noetherian Commutative Ring Theory*, Kluwer Academic Publishers, to appear.

Member of the following Professional Organizations: American Mathematical Society (1981-present), Mathematical Association of America (1985-present), Fulbright Association (1995-present).

Co-organized four different special sessions in Commutative Algebra at Meetings of the American Mathematical Society.

Trinity University Faculty Representative to the National Collegiate Athletic Association, 1990 - 1999. Elected to Executive Committee of the Faculty Athletics Representatives Association (1994 - 1996).

**Publications Closely Related to the Project:**

1. S.T. Chapman and W. Thill*, On a generalization of a theorem of Zaks and Skula, *Proc. Royal Irish Aca.* **96A**, pp. 79–83, 1996.

2. S.T. Chapman, J. Herr* and N. Rooney*, A factorization formula for class number two, *J. Number Theory* **79**, pp. 58–66, 1999.

3. S.T. Chapman, V. DeLorenzo* and H. Swisher*, On The Asymptotic Behavior of Irreducibles in Block Semigroups, to appear in *Semigroup Forum*, `http://www.trinity.edu/schapman`.

4. D.F. Anderson, S.T. Chapman, F. Inman*, and W.W. Smith, Factorization in K$[x^2, x^3]$, *Arch. Math.* **61**, pp. 521-528, 1993.

5. D.D. Anderson, D.F. Anderson, S.T. Chapman and W.W. Smith, Rational elasticity of factorizations in Krull domains, *Proc. Amer. Math. Soc.* **117**, pp. 37–43, 1993.

**Note:** Undergraduate student authors indicated by an $*$.

**Other Significant Publications**

1. S.T. Chapman and W.W. Smith, Factorization in Dedekind domains with finite class group, *Israel J. Math.* **71**, pp. 65–95, 1990.

2. S.T. Chapman, J-L. Chabert and W.W. Smith, The Skolem property in rings of integer-valued polynomials, *Proc. Amer. Math. Soc.* **126**, pp. 3151–3159, 1998.

3. S.T. Chapman and W.W. Smith "Generators of maximal ideals in the ring of integer-valued polynomials," *Rocky Mountain J. Math.* **28**, pp. 95-105, 1998.

4. S.T. Chapman, U. Krause and E. Oeljeklaus, Monoids determined by a homogenous linear Diophantine equation and the half-factorial property, *J. Pure Appl. Algebra* **151**, 107–133, 2000.

5. D.F. Anderson, S.T. Chapman and W.W. Smith, Factorization sets and half-factorial sets in integral domains, *J. Algebra* **178**, pp. 92–121, 1995.

**Collaborators over the Past 48 Months:** D.D. Anderson (Univ. of Iowa), D.F. Anderson (Univ. of Tennessee at Knoxville), P-J. Cahen (Universitè d'Aix-Marseille III, Marseille, France), J-L. Chabert (Université de Picardie, Amiens, France), J. Coykendall (North Dakota State Univ.), V. DeLorenzo (Univ. of Illinois at Urbana/Champaign), M. Freeze (Univ. of North Carolina at Wilmington), J.I. García-García (Universidad de Granada, Granada, Spain), P.A. García-Sánchez (Universidad de Granada, Granada, Spain), S. Glaz (Univ. of Connecticut), A. Geroldinger (Karl-Franzens-Universität, Graz, Austria), F. Halter-Koch (Karl-Franzens-Universität, Graz, Austria), J. Herr (Univ. of Michigan), U. Krause (Universität Bremen, Bremen, Germany), J. Kwak (Kyungpook National University, Ta Gu, Korea), K.A. Loper (Ohio State Univ. at Newark), E. Oeljeklaus (Universität Bremen, Bremen, Germany), K. Roegner (Eckerd College), N. Rooney, J.C. Rosales (Universidad de Granada, Granada, Spain), W.W. Smith (Univ. of North Carolina at Chapel Hill), H. Swisher (Univ. of Wisconsin at Madison), W. Thill, M. Zafrullah (Idaho State Univ.)

**Thesis Advisor:** Nick H. Vaughan (University of North Texas), retired.

**Saber Elaydi**

Biographical Sketch

Trinity University
Department of Mathematics
715 Stadium Drive
San Antonio, Texas 78212-7200
e-mail: selaydi@trinity.edu

**Education:**

Ph.D. in Mathematics, University of Missouri, Columbia 1978.

**Employment:**

Professor, Trinity University, 1999-present

Professor and Chairman, Trinity University, 1991-1999.

Visiting Professor, University of Florence, 1993.

Associate Professor, Trinity University, 1989-1991.

Associate Professor, University of Colorado at Colorado Springs, 1984-1989.

Visiting Assistant Professor, Case Western Reserve University, 1983-1984.

Assistant Professor, Kuwait University, 1978-1983.

**Synergistic Activities**:

1. Instructor of the Year Award, College of Engineering and Applied Sciences, University of Colorado, 1988.
2. Co-Editor-in-Chief, Journal of Difference Equations and Applications, Gordon & Breach, 1994.
3. Member of the editorial board of Journal of Computational and Applied Analysis, 1998.
4. Member of the editorial board of the Journal of Nonlinear Differential Equations, Theory, Methods and Applications, 1997.
5. Editor for the Book series "Discrete Mathematics and Applications", Gordon & Breach, 1995.

**Publications Most Closely Associated With The Grant:**

1. S. Elaydi, *Discrete Chaos*, Chapman Hall/CRC 1999.
2. S. Elaydi, *An Introduction to Difference Equations*, Springer-Verlag, New York, second edition (1999).
3. S. Elaydi, Asymptotic for linear difference equations, J. Difference Equations and Applications, Vol. 5, (1999), 563-589.
4. S. Elaydi, Difference Equations in Combinatorics, Number Theory, and orthogonal polynomials, J. Difference Equations and Applications, Vol. 5, (1999), 379-392.
5. S. Elaydi, Asymptotic Equivalence for difference equations with infinite delay, J. difference Equations and Applications, Vol. 5 (1999), 1-23.

**Other Significant Publications:**

1. S. Elaydi, Periodicity and Stability of linear Volterra Difference Systems, J. Math Analysis & Appl. 181 (1994), 483-492.
2. F. Dannan and S. Elaydi, Lipschitz stability of nonlinear systems of differential equations II: Liapunov functions, J. Math. Anal. Appl., 143 (1989), 517-529. (with Dannan)
3. S. Elaydi, Transformation groups of strong characteristic O, Bulletin Austral. Math. Soc., 27 (1983), 243-248.
4. S. Elaydi, On some stability notions in topological dynamics, J. Differential Equations, 47 (1983), 23-34.
5. S. Elaydi, Criteria for regionally recurrent flows, Proc. Amer. Math. Soc., 85 (1982), 461-468.

**Collaborators over the Past 48 Months:** F. Dannan (University of Damascus), P. Lui (Flinders University, Adelaide, Australia), M. Awartani (Bizeret University, West Bank), S. Murakami (Okayama University, Okayama, Japan), E. Kamiyama (Okayama University, Okayama, Japan), G. Papashinopoulos (Democritus University, Xanthi, Greece), J. Schinos (Democritus University, Xanthi, Greece), K. Janglajew (University of Białystok, Białystok, Poland).

**J. Roberto Hasfura-Buenaga**

Biographical Sketch

Department of Mathematics
Trinity University
San Antonio, TX 78212
Tel (210)999-8240 Fax (210)999-8264
e-mail:jhasfura@trinity.edu

### Education

PhD Mathematics, Wesleyan University, Middletown, CT 1991

MS Structural Mechanics, École Nationale des Travaux Publics de l'État, Lyon, France, 1981

BSCE, Universidad Nacional Autónoma de México, Mexico City, Mexico, 1979

### Professional Experience

Trinity University, Department of Mathematics,
Assistant Professor, 1990-1996
Associate Professor, 1996-Present
Department Chair 1999-Present

Wesleyan University, Department of Mathematics,
Instructor, 1985-1990

Universidad Nacional Autónoma de México, Facultad de Ingeniera,
Instructor, 1981-1982

Universidad Nacional Autnoma de México, Intituto de Ingeniera, Seismology Department,
Research Assistant, 1978-1979

### Publications

1. The Equivalence Theorem for Zd-Actions of Positive Entropy, The Journal of Ergodic Theory and Dynamical Systems, 12, 725-741, 1992
2. Mixing for Dyadic Equivalence, Acta Mathematica Universitatis Comenianae, Vol. LXIV, 1(1995), pp. 141-152
3. Dyadic Changes to Completely Positive Entropy, with Adam Fieldsteel, Transactions of the American Mathematical Society Volume 350, Number 3, March 1998, 1143-1166
4. Periodic Points of the Tent Map, with Phillip Lynch, submitted
5. Asymptotics of Difference Equations, in preparation

### Selected Presentations

1. University of Massachusetts, Amherst, MA, October 1990, 860th Meeting of the American Mathematical Society
2. Rice University, Houston, TX, December 1991
3. University of Texas, Austin, November 1994, Conference in Ergodic Theory

4. Northeastern University, Boston, MA, October 1995, 903rd Meeting of the American Mathematical Society

5. University of Maryland, College Park, MD, April 1996, Penn State-University of Maryland Biannual Conference in Dynamical Systems

6. University of Texas, Austin, April 1997, Mathematical Physics Seminar

**Selected Prior Grant Support**

Co-investigator, NSF-funded Research Experience for Undergraduates Hosted by Trinity University, 1997-1999

**Allen Holder**

Biographical Sketch

Trinity University Mathematics
715 Stadium Drive
San Antonio, TX 78212-7200
Phone (210)999-8241 office; (210)691-8870 home
e-mail aholder@trinity.edu
web-access http://www.trinity.edu/aholder

### Education

University of Southern Mississippi 1986-1990, Bachelor of Science in Mathematics

University of Southern Mississippi 1992-1993, Master of Science in Mathematics, Thesis: Three Interior Point Methods and Their Performance on Small Dense Problems

University of Colorado at Denver 1994-1998, Ph.D., dissertation: *Sensitivity Analysis and The Analytic Central Path*

### Academic Appointments

Trinity University, 1999-present

### Research Papers

1. Allen Holder, *Designing Radiotherapy Plans with Elastic Constraints and Interior Point Methods*, Trinity University Mathematics, Report #49, 2000, in review.
2. R. Caron, H. Greenberg, and A. Holder, *Analytic Centers and Repelling Inequalities*, CCM No. 142, Center for Computational Mathematics, Denver, CO, 1999, in review.
3. A. Holder and R. Caron, *Uniformly Bounding the Limiting and Marginal Derivatives of The Analytic Center Solutions over a set of Normalized Weights*, OR Letters, vol. 29, pages 49-54, 2000.
4. A. Holder, *Symmetric Systems of Linear Equations*, Encyclopedia of Optimization, 2000.
5. A. Holder, J. Sturm, and S. Zhang, *Analytic Central Path, Sensitivity Analysis and Parametric Linear Programming*, Trinity University, Mathematics Technical Report #48, San Antonio, TX, 1999, in review.
6. A. Holder, H. Greenberg, C. Roos, and T. Terlaky, *On the Dimension of the Set of Rim Perturbations for Optimal Partition Invariance*, SIAM Journal on Optimization, vol. 9, pages 207-216, 1998.

### Synergistic Activities

1. *Designing Radiotherapy Plans with Elastic Constraints and Interior Point Algorithms* is nominated for the INFORMS William Pierskalla Award in Health Applications, 2000.
2. Dissertation is nominated for the Council of Graduate Schools National Award, 2000.
3. Developed two software packages, Radiotherapy optimal Design (RAD) is two dimensional radiotherapy treatment planner, and SLEUTH is a linear programming sensitivity analysis tool.

4. 2000 International Symposium on Mathematical Programming, *Analyzing Interior Point Solutions*, invited session chair and talk.

5. 2000 INFORMS ICS conference, *Linear Programming Sensitivity Analysis: A New Paradigm*, invited tutorial.

**Recent Research Collaborators**

Harvey Greenberg, University of Colorado at Denver, Tomas Terlaky, McMaster University, Canada, Jos Sturm, Maastricht University, Netherlands, Rick Caron, University of Windsor, Canada, Shuzhong Zhang, The Chinese University of Hong Kong, Hong Kong, Kees Roos, Delft University of Technology, Netherlands.

**Dissertation Advisor:** Harvey Greenberg

**Directed Undergraduate Research**

1. *Pruning Radiotherapy Treatment Plans*, Son Quach, 1999-2000, Senior Computer Science and Mathematics Projects, Published in the Proceedings of NCUR 2000.

2. *Conforming Optimal Radiotherapy Treatment Plans to the Step-And-Shoot Procedure*, Cory Wetzel, 1999-2000, Senior Mathematics Project.

3. Undergraduate and High School research projects, Summer NASA Fellowship, 1993, University of Southern Mississippi, project: *Implementing Lemke's Algorithm in the MATLAB environment.*

**Teaching Awards**

1. Alpha Lambda Delta Favorite Professor 2000.

2. Lynn Bateman Memorial Teaching Award 1996, Awarded to the teaching assistant demonstrating the highest teaching quality the previous year.

3. STAR Teacher, Greenville Christian School 1993-1994, Awarded to the teacher most influential to the graduating class' STAR student.

**Vadim Ponomarenko**
Biographical Sketch

Trinity University Mathematics
715 Stadium Drive
San Antonio, TX 78212-7200
Phone (210)999-8241 office; (210)691-8870 home
e-mail vadim@trinity.edu

## Professional Preparation

University of Michigan, Ann Arbor, B.S. in mathematics and computer science, 1992

University of Wisconsin, Madison, M.S. in computer science, 1996

University of Wisconsin, Madison Ph.D. in mathematics, 1999

## Appointments

Trinity University, 1999-present

University of Wisconsin, Madison, 1992-1999

## Publications

1. Vadim Ponomarenko, *Reduction of Jump Systems*, submitted to Journal of Combinatorial Theory, Series B.

## Synergistic Activities

1. Investigator streamlined and improved the student tutoring service for the University of Wisconsin mathematics department.
2. Investigator wrote an online Java-based gateway exam in algebra and trigonometry, which has been administered to numerous classes of calculus students. Plans are currently underway to expand this project.
3. Investigator regularly attends professional meetings to disseminate his work and discuss its broader impacts with a wide audience.

## Collaborators and Other Affiliations

1. Collaborators: Antonio Behn, Scott Chapman, Arthur Hobbs, Allen Holder, Christopher Kribs, John Kuchenbrod
2. Postgraduate Advisors: none
3. Thesis Advisor: Richard Brualdi

**Holly J. Rosson**

Biographical Sketch

Trinity University
Department of Mathematics
715 Stadium Drive
San Antonio, Texas 78212-7200
Office Phone: (210) 999-8242 Department FAX: (210) 999-8264
e-mail: hrosson@trinity.edu

## PROFESSIONAL PREPARATION

B.S. in Mathematics,*Magna Cum Laude,*, Saint Michael's College, Colchester, VT., May, 1992.

M.S. in Mathematics, The University of Vermont, Burlington, VT., May, 1994.
Ph.D. in Mathematics, Dartmouth College, Hanover, NH., June, 2000 (Advisor: Thomas Shemanske).

## APPOINTMENTS

*Assistant Professor*, Trinity University, Department of Mathematics, San Antonio, TX., Fall, 2000-Present.

*Graduate Teaching Fellow*, Dartmouth College, Department of Mathematics, Hanover, NH., 1994-2000.

University of Vermont, Department of Mathematics and Statistics, Burlington, VT., 1992-1994.

## PUBLICATIONS

*"Theta Series of Quaternion Algebras over Function Fields"*, Dissertation, Dartmouth College, 2000.

*"Theta Series over Function Fields",* in review.

## SYNERGISTIC ACTIVITIES

*Teaching Seminar Participant, Dartmouth College Summer, 1997.* Participant in an intensive ten week long seminar which considered educational issues, instruction techniques, and presentation skills. Collaborated in preparing a two week long high school mathematics workshop.

*Computers.* Assisted in an abstract algebra class using the mathematical software package Isetl, 1994.

*Software Engineer* at BF Goodrich Aerospace, Vergennes, VT., 1996-1997.

*Software Tester* for mathematical software at Addison-Wesley Publishing Company, Reading, MA., 1994-1995.

**Budget Explanation (Year 1 of 3)**[9]

| | |
|---|---|
| **A.** Salary, Wages and Fringe Benefits<br>$3,000 stipend for the Program Director (Chapman) and 5 Investigators<br>(Elaydi, Hasfura, Holder, Ponomarenko and Rosson) | $18,000 |
| **C.** Fringe Benefits<br>FICA( $0.0765 \times$ $18,000)+TIAA-CREFF($0.10 \times$ $18,000) | $3177 |
| **E.** Travel<br>$200 each for 4 faculty recruiting trips to target institutions<br>in South Texas outside of the San Antonio area | $800 |
| **F.** Participant Costs | |
| 1. Student stipend[10]: 12 × $2,500 | $30,000 |
| 2. Travel: 12×$300 | $3,600 |
| 3. Subsistence: Room and Board<br>$180 × 7 weeks × 12 students | $15,120 |
| **G.** Other Direct Costs<br>Computer Services: $500 for maintanence of the Program's electronic web site<br>for dissemination of Program Application/Reference Forms and Alumni Contact Program | $500 |
| **H.** Total Direct Costs | $71,197 |
| **I.** Indirect Costs<br>Indirect Costs have been waived by Trinity for Cost Sharing Purposes | $0 |
| **J.** Total Direct and Indirect Costs | $71,197 |
| **L.** Amount of Request | $71,197 |

[9] No increase in the budget will take place during years 2 and 3

[10] Students are not considered employees, so 7.65% Social Security tax is not withheld