Representations of Integers

There are some very interesting and surprisingly difficult problems dealing with the representation of an integer n in the base q, where $q \geq 2$ is a fixed integer. One example is the study of base-q *Niven numbers*, which are defined to be those positive integers n that are divisible by the sum $s_q(n)$ of their base-q "digits." The question of the distribution of the base-q Niven numbers was recently answered by by two independent groups of researchers, who proved that the counting function

$$N_q(x) = \#\{0 \leq n < x \ : \ s_q(n)|n\},$$

satisfies the asymptotic formula

$$N_q(x) = (\eta_q + o(1))\frac{x}{\log x},$$

where η_q is an explicit constant.

By analogy, given a pair of integers $p, q \geq 2$ we can consider the collection of integers n that are simultaneously Niven numbers in both the bases p and q. As above, we aim to study the counting function of these numbers:

$$N_{p,q}(x) = \#\{0 \leq n < x \ : \ s_p(n)|n \text{ and } s_q(n)|n\}.$$

It seems that current techniques, which include estimates of certain exponential sums, should apply to the study of $N_{p,q}(x)$ in the case of multiplicatively dependent bases (i.e. when $\log p/\log q$ is rational), whereas the case of independent bases will likely present new challenges. For example, in the latter situation it is not even known whether or not the much weaker statement $\lim_{x\to\infty} N_{p,q}(x) = \infty$ holds. Problems such as these will be the main focus of this summer's project.