Combinatorics REU Project
Summer 2008

Brian Miceli
Trinity University

Pattern Avoidance and Wilf Equivalence

It is a well-known result that the number of 321 and 312-avoiding permutations of $\{1, 2, \ldots, n\}$ is given by the n-th Catalan number, $\mathcal{C}_n$. One can also show that the generating function for the Catalan numbers is given by

$$C(x) = \sum_{n=0}^{\infty} \mathcal{C}_n x^n = \frac{1 - \sqrt{1 - 4x}}{2x},$$

which is not a rational function.

Recent work has shown that an alternative notion of pattern avoidance over all words from the alphabet $\{1, 2, 3, \ldots\}$, subject to certain statistics, will yield rational generating functions. Moreover, one can give bijective proofs that choosing certain (different) patterns to avoid will give the same generating function, and we call such patterns *Wilf equivalent*.

The goal of this summer project will be to prove some remaining results about the Wilf equivalence of certain patterns, and to generalize the known results to include different alphabets and different notions of avoidance.

Although not necessary, programming skills may be helpful for this project, and students interested in this project may wish to include that information in their application materials.