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Linear differential systems with coefficient matrices that commute with a spectrally separated matrix function

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Abstract: We say that a complex matrix-valued function R on an interval $\mathfrak{I}$ is spectrally separated if there are $k \le n$ complex valued functions $\mu_1, \dots, \mu_k$ such that $\mu_1(t), \dots, \mu_k(t)$ are distinct for all $t \in \mathfrak{I}$ and R can be written as

$$R(t) = P(t)\left(\bigoplus_{\ell=1}^{k} \mu_\ell(t) I_{n_\ell}\right) P^{-1}(t)$$

where $n_1 + \cdots + n_k = n$, $P(t) = \begin{bmatrix} P_1(t) & P_2(t) & \cdots & P_k(t) \end{bmatrix}$ with $P_\ell'(t) = P_\ell(t)U_\ell(t)$, and U_ℓ is a continuous complex $n_\ell \times n_\ell$ matrix function, $1 \le \ell \le k$. We characterize the class $\mathcal{C}(R)$ of continuous $n \times n$ complex matrix-valued functions A such that $R(t)A(t) = A(t)R(t)$ for all $t \in \mathfrak{I}$, and show that solving the system $x' = A(t)x$, $t \in \mathfrak{I}$, $A \in \mathcal{C}(R)$, with such a coefficient matrix reduces to solving k independent systems $y_\ell' = F_\ell(t)y_\ell$, where F_ℓ is a continuous $n_\ell \times n_\ell$ complex matrix function, $1 \le \ell \le k$.

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If time permits we will sketch an analogous theory for linear systems of difference equations.