

Bounds for real solutions to structured polynomial systems

Frank Sottile
Texas A&M University

Abstract: Understanding the real solutions to systems of polynomial equations is a difficult question with many applications. In particular, a non-trivial lower bound is an existence proof for solutions and non-trivial upper bounds give complexity bounds.

The best-known upper bound is due to Khovanskii and is unrealistically large.

Lower bounds are a very recent phenomenon, having arisen with real Gromov-Witten invariants and (separately) with the Wronski map in Schubert calculus. The results, though, are striking: “Most” rational curves of degree d interpolating $3d-1$ real points in the plane are real, and every rational function with only real critical points is real.

In this talk, I will describe this background and then discuss recent work giving upper and lower bounds on the numbers of real solutions to some sparse polynomial systems. This is joint work with Soprunova (lower bounds) and with Bihan and Bates (upper bounds).

February 19th, 2008