# A tour of enumerative results in generalized factor order

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Generalized factor order is a partial order on words that was introduced in 2009 by Kitaev, Liese, Remmel and Sagan. In mathematics, think of a **word** as a finite sequence of elements that come from some set (called an alphabet); and more specifically in our case, a partially ordered set (or poset.) Given any poset $(\mathcal{P}, \leq_P)$, and two words $w = w_1 w_2 \dots w_m$ and $v = v_1 v_2 \dots v_n$, we define generalized factor order on the set of words with elements from $\mathcal{P}$ by saying $w$ is less than $v$ if and only if there is a length $m$ substring of $v$ that is componentwise larger than $w$. If you prefer technical definitions:

$$w \leq v \Leftrightarrow \exists i \in [n - m + 1] \text{ such that } \forall j \in [m], w_j \leq_P v_{i+j-1}$$

If $w \leq v$ we say that $v$ **embeds** $w$. It turns out that there are many delightful enumeration problems that arise naturally from studying generalized factor order. In particular, if you let $\mathcal{P}$ be the poset of natural numbers under the standard ordering, plenty of questions emerge. For example, one could ask:

- Given a word $w$, how many words of a given length and sum are there that embed $w$?
- Does the above sequence of such words have a nice generating function?
- Are there different words that yield the same answer to the first question?

In this talk I will give a broad overview of the many enumerative results that can be obtained from the study of generalized factor order. The ability to obtain generating functions for some sequences of interest employs the use of finite automata. Many interesting questions on this topic remain open and I will conclude some of my favorite.