Adjusting Radiotherapy Plans

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May 9, 2000

Abstract

A greedy algorithm is introduced that attempts to transform a complicated conformal treatment plan into the step and shoot paradigm. The algorithm operates on an ideal treatment plan that is currently not possible to implement. The modified plan is capable of being administered by current treatment facilities.

Key words: collimation, conformal plans, elementary beams, pencil
AMS subject classification: 90C05,90C51,90C90

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1 Introduction

We begin with a description of the treatment process. Through the use of a CAT scan or a magnetic resonance image, MRI, an image for analysis is obtained. Such an image provides information about the location, size, and density of tumors. A treatment consists of a given number of angles and a given number of radiation levels, that describes how radiation is administered to the patient. The information provided by the image is taken into consideration when initial plans are made. The main goal of the plan is to focus as much radiation as possible on the tumor while avoiding critical structures. Plans that achieve this goal are known as *conformal plans* [5]. Ideally, conformal plans are always used, but linear accelerators that deliver radiation have limitations that do not allow such treatments. The level of radiation emitted by the linear accelerator is constant, and to overcome the uniform radiation levels, radiation passes through a gantry. Within the gantry there are lead bars that are use to shape the radiation beam.

There are two paradigms that control how the treatment is administered at this point. The old paradigm used metal wedges that shield some of the radiation. With this paradigm, conformal plans are usually unattainable because the wedges do not make the intricate beam adjustments necessary for conformal plans. Most current treatment centers have linear accelerators with gantries that contain many metal plates known as leaves. The leaves are adjusted to attain the appropriate radiation levels. The advantage of the multiple leaves as opposed to the wedges is the ability to make very fine adjustments to the beam of radiation delivered to the patient. This is known as beam *collimation* [5]. Currently, the adjustments are made manually in the leaf settings, but soon technology will allow an automated computer process to make the leaf setting changes as the gantry rotates. However, this technology has yet to be fully implented.

Therefore, once conformal treatment plans are devised, the plans must be modified in order to be implemented. Moreover, the adjustments need to be made in such away that the plan adheres to the doctor's prescription. Because the plans must be altered to work in the manual leaf setting environment, several factors need to be considered for plans to be feasible. First, there is a restriction on the number of beams that can be used. This is due to the fact that each beam requires the gantry to be moved into a specific position before radiation can be delivered. Second, the number of leaf settings is restricted. Leaf setting changes are time consuming, and therefore, must be minimized. Maintaining exact conformal plans is therefore currently impossible because the time required for many settings is unreasonable. Most treatment facilities try to keep a patient's treatment time to fifteen minutes because there are so many patients to treat. This is where our problem lies, we must minimize the number of beams and leaf changes while maintaining appropriate radiation levels over the surface of the tumor, and do so in a manner that allows treatment to be effectively delivered in an appropriate amount of time.

2 Plan Adjustments

Because the dose level to a cell is additive with respect to time, a linear operator is used to deposit energy into the patient image. This linear operator is defined by a matrix A. The matrix A consists of rows that contain information about each pixel of the patient image. The columns of the matrix A are divided into sets, these sets describe the number of angles in the matrix A. Each element of the set, which is an individual column of the matrix, represents a *pencil*, or elementary beam of radiation for that particular angle [5]. The pencils are included in our model because modern treatment systems are capable of intricate collimation [2]. A component of the matrix A is represented by $A_{(p,(a,i))}$, where p is the pixel of the patient image, a is the angle of the beam, and i is the individual pencil within angle a. Each element of the matrix that contains information about the tumor area of the patient image is donated as A_t. These entries are positive, and each positive element of the matrix will have an attenuation factor $e^{-\mu d}$, that describes the energy lost by the radiation beam after traveling distance d through the body. An example is depicted in Figure 1.

Our problem begins with an optimal treatment plan, denoted by x. The components of x are $x_{(a,i)}$, where $x_{(a,i)}$ describes the dose of radiation for i^{th} sub-beam of angle a. The vector x given is an optimal treatment plan before modification. This is important because ideally we want to maintain the dosage levels provided by Ax.

More specifically, the tumor dosage must be high enough to ensure that the tumor is killed, but not so high that it destroys good tissue as

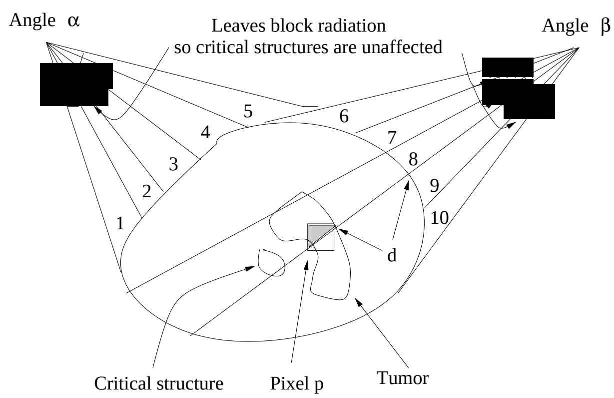

Figure 1: In this diagram $A_t(p, (\beta, 8)) = \frac{2}{5}e^{-\mu d}$, where d is the distance the beam travels through the body to the pixel, and the $\frac{2}{5}$ is the amount of the grey area filled in on pixel p in pencil 8.

well. These conditions are known as the tumor lower bound, denoted TLB, and the tumor upper bound, denoted TUB. This means we want to find some δx such that $A(x+\delta x) = Ax$ in order to stay within the range of the tumor upper bound and the tumor lower bound. This accomplishes our goal nicely because we would be able to change the gantry and leaf settings a specific number of times without affecting the dosage of radiation delivered to the tumor area. Now, with this information we find the problem of minimizing the number of leaf changes to be:

$$\text{Find } L = \{l_1, l_2, \ldots, l_j\} \text{ such that}$$

- j is as small as possible,
- $x_i + \delta x_i \in L$, and
- $A_t \delta x = 0$.

The last condition is overly restrictive. For example, if the null space of A_t is zero this implies that there exists no δx other than zero that satisfies the final condition. Despite this we can guarantee that the plans will not exceed the tumor upper bound if certain conditions are meet. This condition is formally stated in the following theorem.

Theorem 1 *If*

$$\|\delta x\|_\infty \leq \frac{\min_i\{TUB_i\} - \|A_t x\|_\infty}{\|A_t\|_\infty},$$

TUB is guaranteed to be satisfied by the modified plan $x + \delta x$.

Proof: Using the triangular inequality and sub-multiplicity [1] we have,

$$\begin{aligned} \|A_t x + A_t \delta x\|_\infty &\leq \|A_t x\|_\infty + \|A_t \delta x\|_\infty \\ &\leq \|A_t x\|_\infty + \|A_t\|_\infty \|\delta x\|_\infty . \end{aligned}$$

So, if $\|A_t x\|_\infty + \|A_t\|_\infty \|\delta x\|_\infty \leq \min_i\{TUB_i\}$ we have that the modified plan, $x + \delta x$, satisfies the upper bound constant on the tumor. Since this inequality is equivalent to

$$\|\delta x\|_\infty \leq \frac{\min_i\{TUB_i\} - \|A_t x\|_\infty}{\|A_t\|_\infty},$$

the proof is complete. ■

This bound is calculated and used to measure how much we may alter the initial plan. Then, if we also satisfy the TLB, we achieve optimal plans that are implementable. However, achieving this goal is more difficult than it first appears.

Treatment planners are faced with a far more difficult task than just ensuring the tumor receives the correct amount of radiation. Because the radiation kills living cells without prejudice, the treatment planner must be careful when deciding what angles and radiation amounts to use. Work is currently being done in this area to optimize the treatment plan [2, 6, 4]. The problem is threefold.

- The plan must ensure that the tumors receive a minimal dose in order to kill the cancerous cells.
- Plans should ensure that critical structures receive minimal radiation.
- All other good tissue should receive as little radiation as possible. In other words there should be no "hot spots" outside the tumor.

This complicates the problem further because there are new factors to consider. The tumor lower bound ensures that a tumoricidal dose of radiation is delivered to the tumor. However, this goal alone is no longer sufficient to maintain conformal plans. The plans must now take into consideration a critical structures upper bound, or CUB, and a good tissue upper bound, or GUB. These factors make the adjusting of plans far more difficult because any minute change could have an adverse affect in the overall outcome. Therefore, care must be taken when adjusting the plan to ensure that no upper bounds are being exceeded and that the radiation over the tumor is at least TLB.

3 Ideal Program Model

We assume all of the necessary angle pruning is done, thereby removing excess beams. Such procedures have been looked into by Son Quach [4]. The plans provided after the pruning procedure are for continuous leaf setting changes. However, due to limitations in technology the plans must be compatible with the step and shoot process. This means we must discretize the leaf setting changes. There are several ways of discretizing the leaf settings, but our group decided to use a two stage recourse model. This model follows a very simple scheme.

Algorithm Layout

Step 1 Choose a driving beam.

Step 2 Adjust driving beam.

Step 3 Re-optimize with remaining beams.

Step 4 Repeat.

First, the program will search through the matrix A and find the beam that drives the radiation treatment. The program categorizes the beams by column sums. The idea being that the largest column sum is the beam that hits the tumor the most. However, incorporating the plan x is important because it contains more information about the treatment dosage levels. This information is what determines the plans more so than the column sum of the matrix A. So, the model looks for the maximum element in x. The program then compares this information with the maximum column sum. The two pieces of information together determine the driving beam for the plan.

Once the driving beam is selected the program makes the appropriate changes to the dosage levels so that the number of leaf changes for a particular beam is reduced, and able to be done within a reasonable amount of time. For now the prototype uses averages to reassign values to the vector x. Since the number of row elements in x is equivalent to the number of column elements in A, the vector x is easily subdivided into a set that corresponds to the set that makes up the angles in the matrix A. This means that portions of the vector x will be changed based on information from the driving beam of the sub-matrix of A_t. The driving beam is denoted as $\bar{A}_t^j$, which is a sub matrix of A corresponding to the columns found in angle j, and A_t^j contains the remaining columns of A_t. So that now with the angle removed we have $A_t x = A_t^{j_1} x + \bar{A}_t^{j_1} x_{(\cdot,(a_j,\cdot))}$, or in other words we have the the same matrix with one angle removed and adjusted.

Once the angle is removed the plan must be re-optimized with the remaining beams. Ideally we want the changes made in the beam to be optimal. To insure this once the changes have been made to the driving beam a new optimal plan is recalculated, upon subtracting the changed angle from each end of the inequality. Therefore, we have $TLB - \bar{A}_t^{j_1} x_{(\cdot,(a_j,\cdot))} \leq A_t^{j_1} x \leq TUB - \bar{A}_t^{j_1} x_{(\cdot,(a_j,\cdot))}$ as our new inequality for the procedure. This process of removing angles, adjusting, and re-optimizing is repeated until all of the beams have been changed in an appropriate fashion. We use the MATLAB© linear programming solver to analyze the practicality of our process. Essentially, the linear program tests the difference between our new dose deposition matrix and our original to ensure that the plans still follow the prescribed radiation dosage levels [2]. This means our program ensures that none of the bounds are violated.

4 Conclusions

The above described algorithm still requires work. The method of readjusting the values of x have not been fully explored. For now the algorithm works in a primitive fashion, and is able to adjust the vector x in the manner described, but the optimization of the plan is frequently unattainable. This is due to some of the limitations in the software package, but more likely using averages to readjust the vector x introduces extraneous errors. Hopefully this can be overcome in the future so that all plans are able to be optimized and implemented.

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