Pruning Radiosurgery Plans

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Abstract

Because radiation plans automatically generated by optimization routines are often too complicated to implement, we have developed a method to prune excess angles. A plan is generated using linear programming and is then pruned to obtain the desired number of angles using several different merit functions. Plans are pruned based on tumor uniformity, critical structure dose, and a combination of the two. We conduct several experiments and discuss our results.

Key words: radiation therapy, treatment planning, optimization, linear programming
AMS subject classification: 90C05, 90C90

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1 Introduction

External beam radiotherapy is the treatment of inoperable tumors with radiation. While in the past, treatment planning was done by a person on a trial and error basis, modern technology has created a newer generation of machines that make planning more complicated. An optimization routine chooses a collection of radiation beams from a prescribed set of possible beams which pass through a patient at certain angles [1, 3]. The specific choice of angles and intensities is called a *plan*. We desire to find treatment plans that satisfy several goals: those that do not have hotspots, allow for a uniform dose over the tumor, and keep radiation to critical structures at a minimum.

An optimal plan found using linear programming usually has too many angles for clinical use. However, newer treatment facilities are capable of implementing these complicated plans. Older clinics require a method to *prune*, or remove, excessive angles to obtain a practical plan. The pruned plan is no longer optimal but is feasible for the majority of clinics. An acceptable plan usually contains between 3 and 7 angles and should strive to achieve the desired goals. A pruning process has been attempted in the past with limited success [2]. It is necessary to use computer automated plan designs for this because of the size of the problem. The newer method discussed below appears promising.

2 Radiotherapy

Cancer patients are treated with a machine called a linear accelerator, see Figure 1. The patient lies on the couch as the linear accelerator creates a high energy beam that passes through the gantry (the head of the linear accelerator) which revolves around the patient. The beam of radiation is shaped by blocking a certain amount of the radiation. Radiation is deposited into cells as the beams pass through the patient. Each beam consists of sub-beams called *pencils*. The pencils fan out from a point source, and each pencil may have different radiation weights due to *collimation*. A multileaf collimator can block any portion of the beam as the beam passes through it.

Figure 1: A linear accelerator

A treatment plan consists of the collection of beam angles and their beam weights A desirable treatment plan consists of a high enough level of radiation to kill cancerous cells. used to treat a patient. A desirable treatment plan has almost all the high radiation levels within the tumorous regions.

Newer facilities allow for more beams to be used. With more beams, each angle intersecting the tumor has a lower level of radiation, where the accumulative dose remains high enough to kill the cancerous cells. A one-beam plan is very unlikely to be optimal because this means there is just one high level of radiation passing through the patient, which would have adverse effects on much of the nontumorous tissue. Most treatment plans involve from three to seven beams at older facilities, where radiation oncologists usually experiment with the angles and beam intensities manually to find a plan that looks good. This allows for human error. Also, as larger and more complicated treatment plans become implementable, there are too many angles to consider this way. Instead, we propose to use a linear program to design a plan that uses many angles and then prune the plan so that the number of angles is decreased to some suitable threshold. We conjecture that these plans are better than those found by a radiation oncologist.

3 The Model

We start by describing the *dose deposition matrix*, A. The components of A give the percentage of a pencil that will hit a corresponding pixel of the patient image. The rows of A are divided into those that correspond to the tumorous tissue and those that correspond to *critical structures* (nontumorous tissue that is sensitive to radiation). The partitioning of A is depicted by

$$A = \begin{bmatrix} A_{Tumor} \\ A_{Good} \end{bmatrix}.$$

Figure 2 shows a simple example of a 2×2 grid with 4 equally spaced angles, each consisting of 3 *elementary beams*. The matrix A will have 4 rows, the number of pixels in the grid, and 12 columns, the number of angles times number of sub-beams:

$$A = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}.$$

In general the size of A depends on the number of angles, a, the number of pencils, p, and the $n \times n$ grid size. The number of rows of A is $n \times n$ and the number of columns is $a \times p$.

We now define the following vectors:
TLB - lower bound values for tumor radiation dosages,
TUB - upper bound values for tumor radiation dosages,
GUB - upper bound values for critical structure dosages, and
TC - the number of pencils that hit the tumor.

The vectors TUB, TLB, and GUB are collectively called the *prescription.* GUB is initially set to be the minimum level of radiation that causes permanent harm to the critical structures. TUB and TLB are found by setting a desired radiation dosage for the tumor in addition to a uniformity level. The uniformity level is a percentage that is added and subtracted to the tumor goal from which we obtain TUB and TLB. For example if the tumor goal is 80 and the uniformity level is set at 5%, then TUB is 84 and TLB is 76. More angles, more pencils, and a larger grid size usually allow better plans because there is more detail and choices. The mathematical model solved to find a treatment plan is:

$$\min \omega e^T \alpha + e^T \beta + z + \left(\tfrac{1}{TC}\right) e^T x$$

subject to

$$TLB - \alpha \leq A_{Tumor} x \leq TUB$$

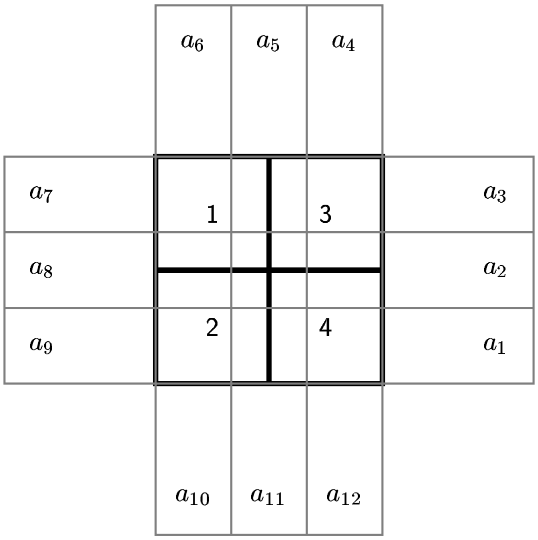

Figure 2: The geometry of a 2 × 2 pixel image with 4 angles

$$
\begin{gathered}
0 \leq A_{Good} x \leq GUB + \beta \\
0 \leq x \leq ze \\
0 \leq \alpha \leq TLB \\
-GUB \leq \beta \\
0 \leq x,
\end{gathered}
$$

where

ω - chosen to guarantee tumor uniformity,
α - tells how deficient a plan is with respect to TLB,
β - explains any discrepancy from GUB,
z - the highest possible dose, and
x - vector of radiation dosages called the plan.

Our decision variables are ω, α, β, z, and x. If ω is large enough, the linear program will drive α to 0 assuring that $A_{Tumor} x$ is within TLB and TUB [1]. There are three goals for this model. The first goal is to guarantee tumor uniformity. The next goal is to minimize radiation dosage to the critical structures. This occurs because the model tries decrease β to $-GUB$, meaning there is no radiation going to the critical structure. The final goal is to minimize the radiation from any single pencil and to minimize the total radiation dosage. This model gives an optimal radio surgery plan with respect to the objective function. Optimality also depends on the constraints set and the type of solution technique employed.

4 Pruning

Pruning is necessary in order to obtain plans that are useful in most treatment facilities. Powlis attempted pruning in the past using a method that assumed that the beams with the largest beam weights should be kept [2]. Our algorithm looks at all beam angles to find the one with the least affect on the plan and removes it. We propose a greedy algorithm for the pruning process. The optimal plan is compared to all possible plans with one angle removed. The one that has the least effect when removed is the angle that is pruned, see Figure 3.

The comparison step relies on a merit function that is based on minimizing the effect of radiation to the tumor, to the critical structure, or a combination of the two. The merit

A Greedy Algorithm for Pruning

Step 0. Remove any angle that had total radiation less than 10^{-2}.

Step 1. Compare the optimal plan to another optimal plan with an angle removed based on one of the merit functions. Prune the angle with the least effect on the original optimal plan.

Step 2. Repeat Step 1 until the desired number of angles is obtained.

Figure 3: The Greedy Algorithm

function based on tumor is

$$||A_{Tumor}x - A^i_{Tumor}x^i||,$$

where A^i is the submatrix of A with angle i removed. The merit function based on critical structures is given by

$$||e^{\beta^i}||_2,$$

where β^i is found with angle i removed. Here, e^{β^i} is the exponential function applied to each component of the vector. A combination of both tumor and critical structure is

$$\frac{||A_{Tumor}x - A^i_{Tumor}x^i||||e^{\beta^i}||}{||A_{Tumor}x||||e^{\beta}||}.$$

This is a combination of the first two merit functions that is normalized. These different merit functions give results that differ as more pruning is done.

As an example, consider an optimal plan that uses 24 equally spaced angles. Suppose that 10 of those angles had radiation less than 10^{-2}. These angles would be removed in Step 0. This leaves 14 remaining angles. In Step 1, the optimal plan with 14 angles is compared to each optimal plan with 13 angles. The one with the least effect is then removed. Angles are removed one at a time until a feasible plan is achieved.

5 Experiments

The experiments used a 64 by 64 grid. There were 24 angles and each angle contained 10 pencils. We decided on these numbers because they could be solved in a reasonable amount of time using MATLAB. Facilities normally have a 512 by 512 grid, 360 angles and 124 pencils. MATLAB and its Optimization Toolbox were used. The results can be seen in figures 4-27. The merit functions based on tumor and critical structure return different results. Unfortunately, we obtain the same results for the tumor merit function and combination merit function. We hope to find a better combination merit function. An experienced physician should examine the results to determine which merit function is best for each individual patient image. The plans pruned down to 2 angles will probably never be used because there are too many hotspots. Plans pruned down to 6 angles look feasible and show that the pruning process is promising.

6 Discussion

Radiation treatment therapy planning is changing due to newer machines that are capable of handling a plan with many angles. For the older facilities, plans can only contain a few angles. This gives a plan that is suitable but not optimal. Different merit functions were used to perform well for a wide variety of patient images. We would like to find other merit functions that better describe physician's desires.

In the future, we hope to extend our experiments to larger plans using the same grid size, angles, and pencils as the facilities use. This would entail using a commercial solver such as CPLEX or PCx. Also, we would like to extend to 3-D treatment planning. This gives more angle choices and treatment options.

References

[1] Allen Holder. Designing radiotherapy plans with elastic constraints and interior point methods. April 2000.

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[3] David M. Shepard, Michael C. Ferris, Gustavo H. Olivera, and T. Rockwell Mackie. Optimizing the delivery of radiation therapy to cancer patients. *SIAM Review*, 41(4):721–744, 1999.

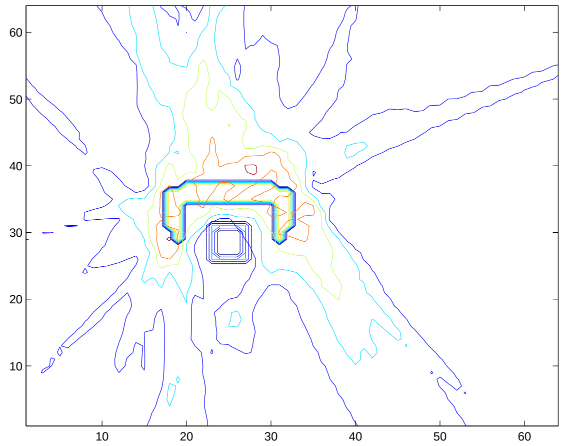

Figure 4: The optimal plan with 24 angles. The patient image is a brain stem surrounded by a tumor.

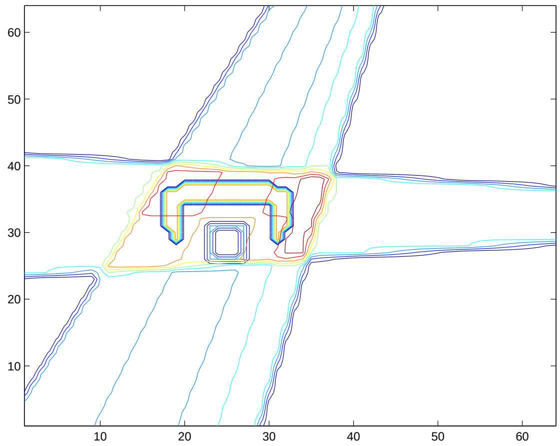

Figure 5: The tumor merit function was used to prune to 2 angles.

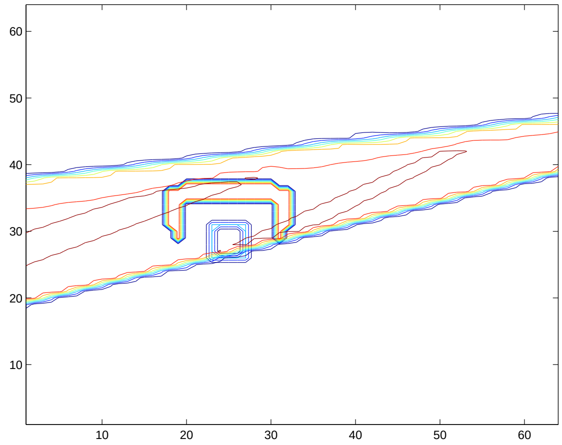

Figure 6: The critical structure merit function was used to prune to 2 angles.

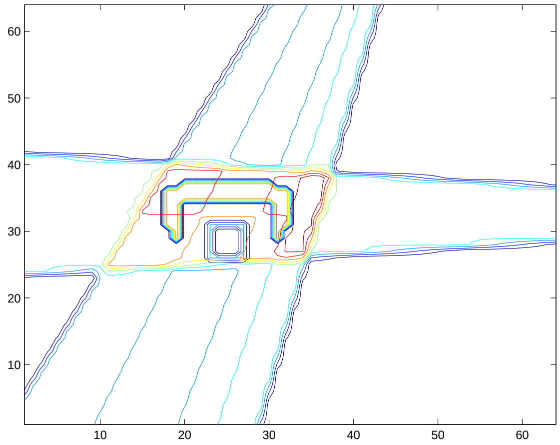

Figure 7: The combination merit function was used to prune to 2 angles.

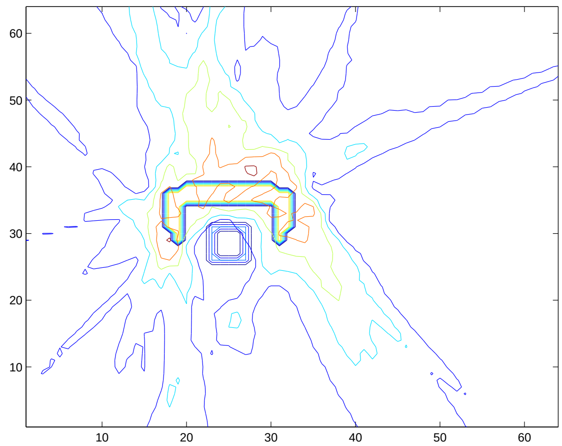

Figure 8: The optimal plan with 24 angles. The patient image is a brain stem surrounded by a tumor.

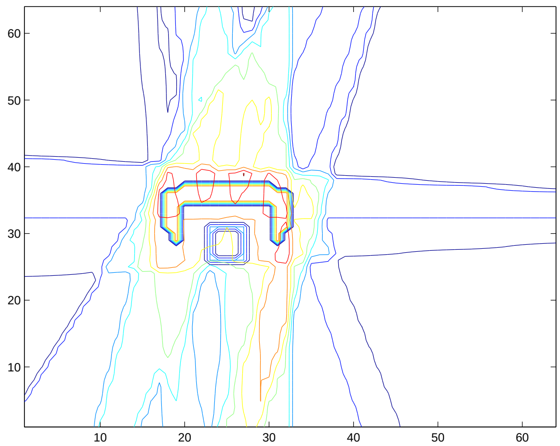

Figure 9: The tumor merit function was used to prune to 4 angles.

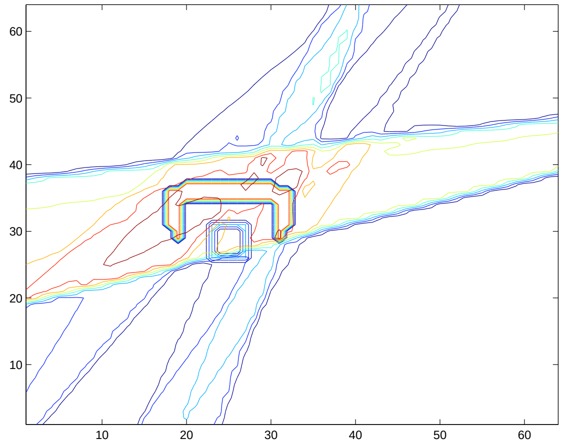

Figure 10: The critical structure merit function was used to prune to 4 angles.

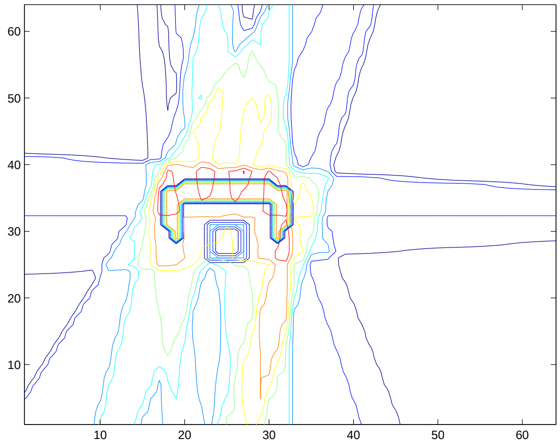

Figure 11: The combination merit function was used to prune to 4 angles.

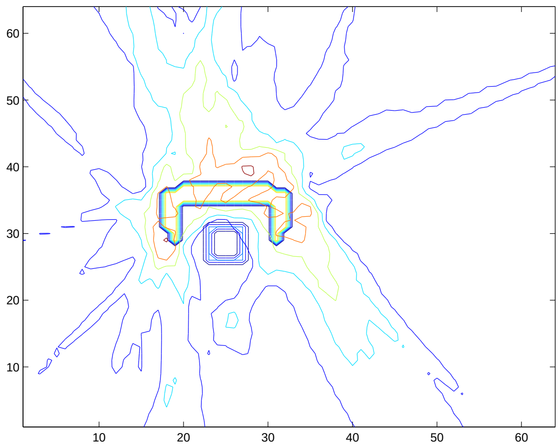

Figure 12: The optimal plan with 24 angles. The patient image is a brain stem surrounded by a tumor.

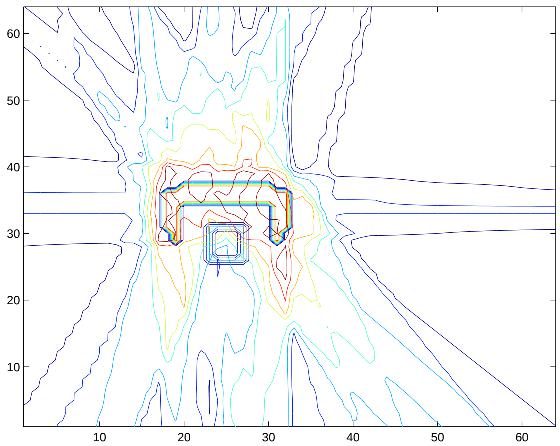

Figure 13: The tumor merit function was used to prune to 6 angles.

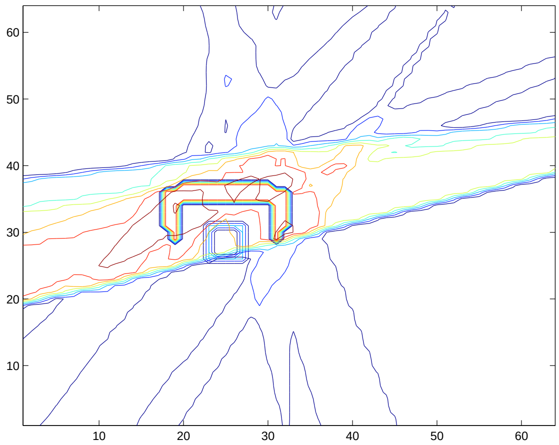

Figure 14: The critical structure merit function was used to prune to 6 angles.

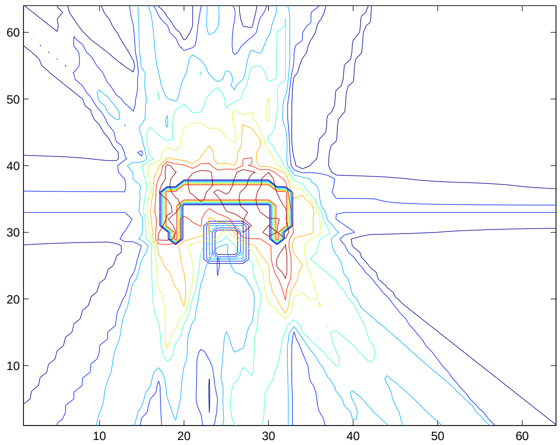

Figure 15: The combination merit function was used to prune to 6 angles.

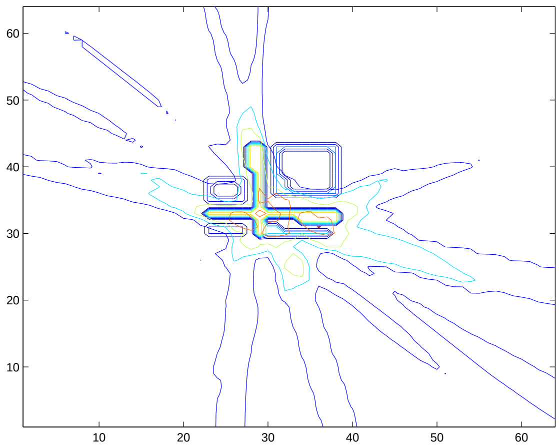

Figure 16: The optimal plan with 24 angles. The patient image contains a tumor surrounded by four critical structures.

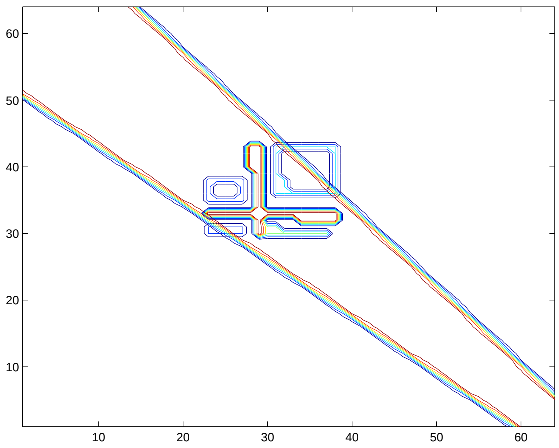

Figure 17: The tumor merit function was used to prune to 2 angles.

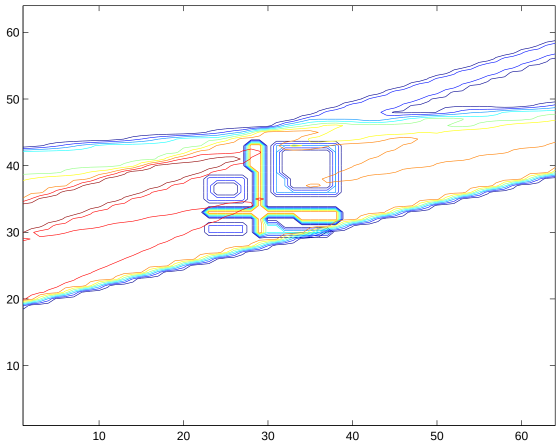

Figure 18: The critical structure merit function was used to prune to 2 angles.

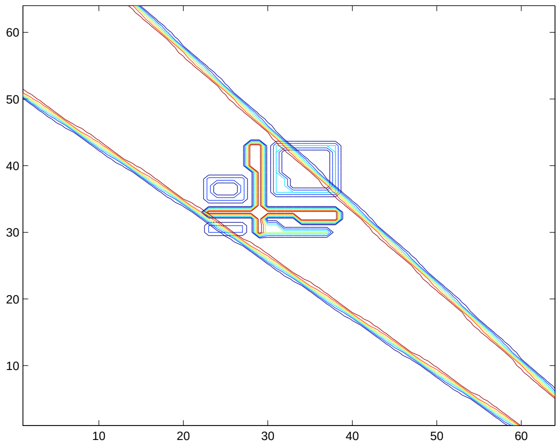

Figure 19: The combination merit function was used prune to 2 angles.

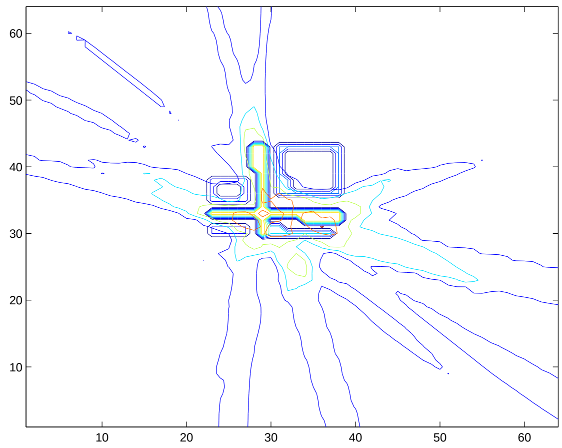

Figure 20: The optimal plan with 24 angles. The patient image contains a tumor surrounded by four critical structures.

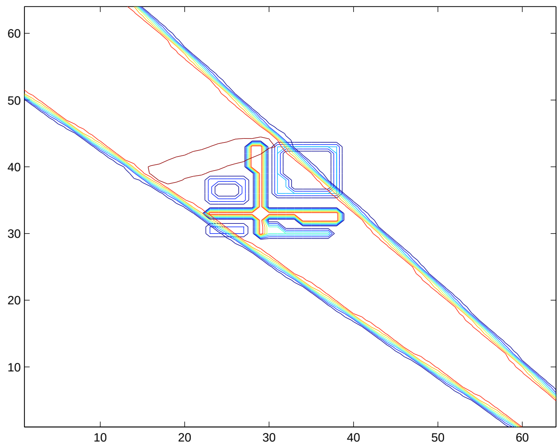

Figure 21: The tumor merit function was used to prune to 4 angles.

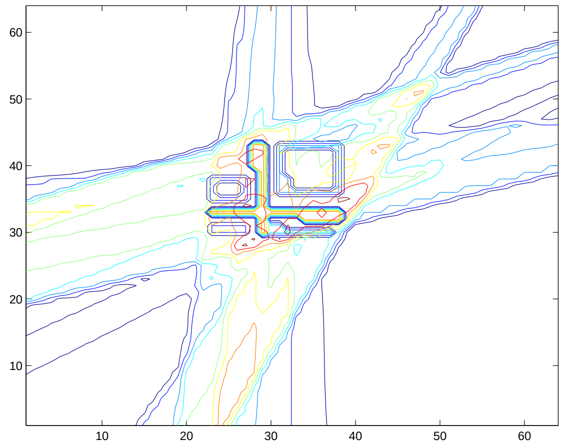

Figure 22: The critical structure merit function was used to prune to 4 angles.

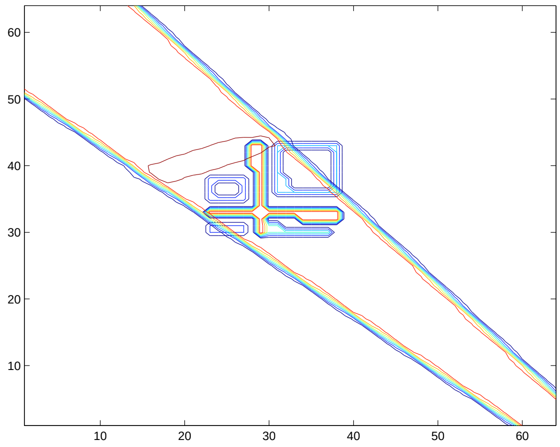

Figure 23: The combination merit function was used to prune to 4 angles.

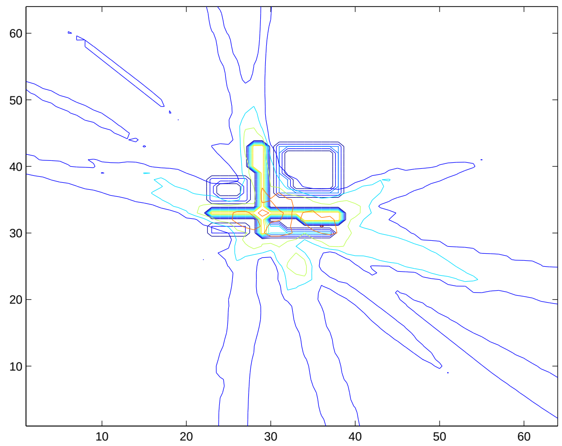

Figure 24: The optimal plan with 24 angles. The patient image contains a tumor surrounded by four critical structures.

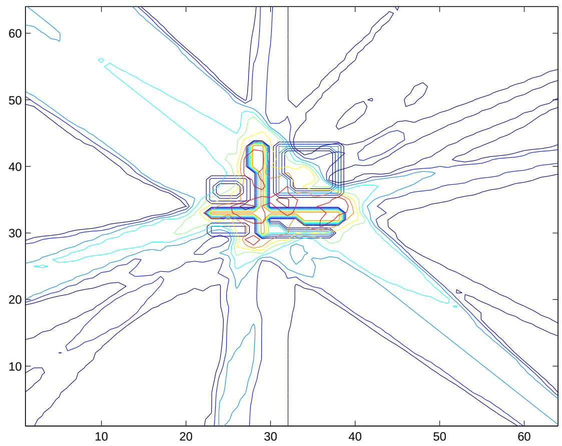

Figure 25: The tumor merit function was used to prune to 6 angles.

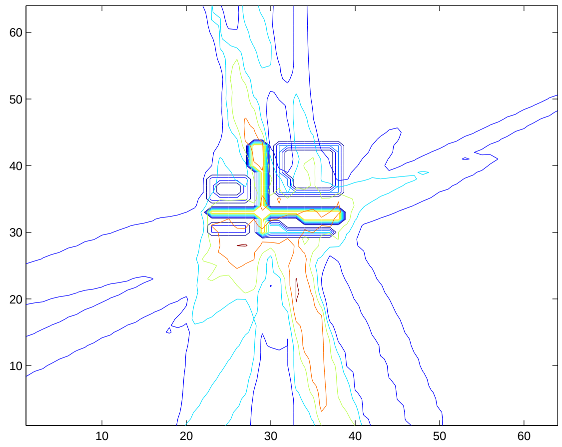

Figure 26: The critical structure merit function was used prune to 6 angles.

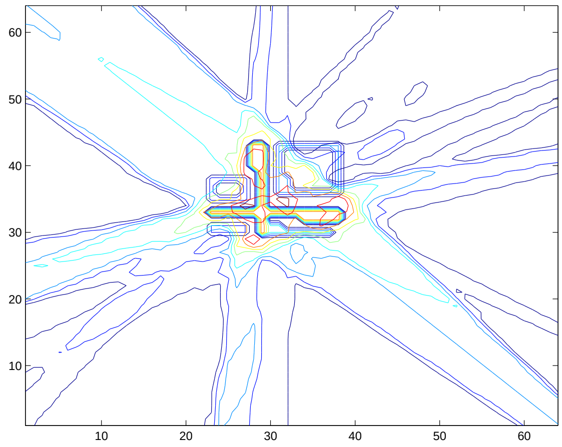

Figure 27: The combination merit function was used to prune to 6 angles.