An Honors Junior High Course in Social Mathematics

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1 Introduction

What fate is to befall the junior high mathematics student? Is he or she to be expected to embrace an ever-quickening road directly through the algebra and geometry sequence to a calculus class that is now often taken (gasp) in the junior year in high school? Or will he or she be left behind the front, remaining forever at a disadvantage in a world dominated by those students whose natural advantages have enabled them to travel efficiently along this road? Or is there some other option?

Actually, there are many other options. Students have increasingly been using the summer to expand their educations in a plethora of exciting directions. The goal of this study was to produce exactly this: the teaching materials that can form the corner stone of a summer experience for junior high students interested in an alternative direction in their development in mathematics; henceforth, we will label this course the Passty Course. Presented during the course of this paper will be how the author's personal history led him to dream of such a course. Next, the goals of the project will be presented. Then, a through discussion of the market into which this course will enter will be given. After spending considerable time developing and presenting the product itself, directions for future development will be presented.

2 Personal History of the Author

As the wedding of mathematics and education, this project perfectly reflects my own background both as a mathematician and as an educator. As the son of a professional mathematician and mathematics major at Trinity University, my exposure to traditional undergraduate mathematics has been quite extensive. However, this pedigree has also resulted in a tremendous exposure to social mathematics beginning at an early age. By the age of 13, I was benefitting from my father's explanations of his new liberal arts mathematics class that he was developing at Southwest Texas State University. By the age of 16, I had competed in State Science Fair with my own project that was a computer simulation of the famous Monty Hall Dilemma that had already stumped some of the greatest mathematical minds in the country.

My experiences as a mathematics major at Trinity have been no different, as I have found room in my schedule for seminars in Game Theory and Financial Mathematics to supplement my more traditional studies.

However, my involvement with education was the true source of interest in this project. Whether as an instructors' assistant at the Sylvan Learning Center, or a Peer Tutor, or even the Math Department Help Sessions Coordinator, my college experience has been rich in opportunities in help other people learn about mathematics. The love of teaching shown by these various activities was what truly impelled me to develop the Passty Course. It is my hope that—throughout the discussion contained in this statement—that love of teaching will be made obvious to the reader as well.

3 The Goals

There were three main goals of this project:

1. Create a multi–week experience in which students are exposed to non–traditional mathematics
2. Develop a student's skills in abstract reasoning and problem solving
3. Present material that students may use for a lifetime of decision–making

3.1 Creating a Multi-Week Math Experience

The most important priority of the project was to create the lesson plans that would become the basis of a several week course in mathematics. Satisfaction of this goal would be difficult, as enough planning will have to happen to provide the tools for the teacher. However, as Dartmouth has shown in their Chance course, it would be difficult indeed to plan every detail of such a course without a foreknowledge of the current events that would happen in the world during the teaching of the course. This was a lesson that I learned myself on November 8, 2000, when I woke up expecting to present my "Now Gore has lost: What do We Do?" lesson to economics students at Reagan High School only to find out that the premise of the lesson hadn't actually happened.

Nevertheless, reasonable expectations of the plans can be formed. The principle unit of thought for this project became the concept plan. Thanks to my training under Prof. Richard Butler through the Economists in the Schools Program, I already had great deal of skill in preparing such plans, one each of which was centered around a major topic of the course. Together, these concept plans contain enough mathematical material to amuse both instructor and students for a long time. More importantly, the mathematics provide a brilliant spring board into a discussion of issues relating to the news or to our daily lives.

3.2 Developing Abstract Reasoning and Problem Solving Skills

This goal was outlined by Manuel Berriozabal in his January 2002 speech to the Trinity University Math Majors' Seminar. The goal of any branch of mathematics is problem solving, and the Passty Course is intended to train students in order to do this, both through subject and method.

Problem solving through method is exhibited in an approach that involves reducing unsolvable problems into problems that have previously been solved. Students are urged to build a literal "toolbox" of techniques for their use. For example, in calculus this toolbox could include such facts as $\sum_{i=1}^{n} i^2 = \frac{n*(n+1)}{2}$. Once this fact is known, then the student has gained access to sums such as $\sum_{i=1}^{n} i^2 + 3$ or $\sum_{i=1}^{n} 3i^2$, or even $\sum_{i=14}^{n} i^2$.

Problem solving through material is accomplished through selecting topics that lend themselves to open-ended discussion. The topics of human behavior featured in Social Choice and Game Theory lessons are quite useful in that real life examples often feature decision–makers behaving in a manner that diverges greatly from that predicted by economic theory.

3.3 Material that Students May Use for a Lifetime of Decision–Making

An even greater stength of the material is that it relates to issues that students can expect to think about for a lifetime. There was not a person in the United States who did not at least ponder the continued use of the electoral college system after the most recent Presidential election. The Passty course trains students in the mathematics needed to consider decisions as innocent as the purchase of a lottery ticket, while explaining issues as grievous as the two year delay in San Marcos, Texas, in repairs to the dam that was damaged during the great flood in October 1998.

Equally important in arming the next generation for lifetime use of mathematics is granting them the right to be skeptical. The rate of information flow today is so great that we can never hope to know all of it; therefore, we rely upon statistics. Unfortunately, "the secret language of statistics, so appealing in a factminded culture, is employed to sensationalize, inflate, confuse, and oversimplify [**?**]. Equipping our students to sift through the noise to the facts should be a high priority among educators.

4 The Alternative Math Program Market

The Passty course enters into a market that is diverse both on the supply– and demand–sides. Many exciting programs are already in existence; some are presented below:

4.1 Current Programs

Institution	Program	Contact Person
UTSA	San Antonio Prep Program	Manuel Berriozabal
SWT	Mathworks Honors Summer Math Camp	Max Warshauer
UT Austin	Emerging Scholars Program	Harrison Keller
Johns Hopkins	Center for Talented Youth	Lea Ybarra
Dartmouth	Chance Course	Peter Doyle

The Chance Course is widely recognized as the cutting edge figure in the market for Math Summer experiences; it consists of a series of innovative seminar lectures, discussions, and problem solving sessions held at Dartmouth each summer. No similar course could be planned without paying the proper homage to the Chance course's treatment of exciting Social Mathematics and development of young minds.

4.2 Current Students

In general, students who participate in programs such as this one are destined for much success. According to Manuel Berriozabal, 93% of students who participate in the PREP program at UTSA eventually graduate from college.

Most students involved in summer math enrichment programs have one of two goals:

1. To accelerate their progress throught the traditional math sequence in algebra, geometry, and calculus, or

2. To tap enrichment opportunities orthogonal to this traditional math sequence.

While it does assume a basic familiarity with algebra and algebraic operations, the Passty course is concerned entirely with goal #2. This is due more to the interests of the author in exotic mathematics than to any other factor.

5 The Product

The format of the Passty course consists of 11 concept plans—developing the themes of probability, game theory, social choice, and allocation—and 2 technology lessons over use of a graphing calculator and Microsoft Excel.

5.1 Mathematics Material

The concept plans are intended to focus students to acquire the tools necessary to understand four problems.

The Monty Hall Dilemma: Suppose that you are looking at doors numbered 1 through 5. Monty Hall tells you that he has placed goats behind four of the doors and a new car behind one of them. "First you select two doors," he explains. "Then, I will open two of the other doors and show you goats behind them. Finally, you will have the opportunity to either keep your original two doors or switch to the remaining unopened door. If you have a car behind your door (or either of your two doors), then you win the car. If you have only goats behind your doors, you leave with nothing." What is a good strategy for maximizing your chances of winning the car?

The solution to this problem involves using tools of conditional probability. We first consider the strategy of always staying with the initial two–door guess. There are five doors. Therefore, there are $\binom{5}{2} = 10$ possible pairs of doors. Out of these ten pairs, four will have the car in them. Therefore, the strategy of never switching has a winning percentage of 40%. So always switching, even though we seem to be moving from 2 doors to only 1, will win 60% of the time.

This problem is a very difficult problem with regard to understanding, as the solution is counter–intuitive. Many notable mathematics Ph.D.'s have been stumped by it (there was quite a larger fervor over it begun by Marilyn vos Savant begun back in the late 1980's)[**?**]. However, this counter–intuitive nature makes it perfect for a course such as the Passty Course, as junior high students will benefit tremendously from such problems, which force them to use mathematical tools rather than more elementary reasoning.

The Iterated Prisoner's Dilemma: Consider the classic problem of the Prisoner's Dilemma. In matrix game form, it can be represented as:

		Player B	
		Cooperate	Defect
Player A	Cooperate	$(3,3)$	$(0,4)$
	Defect	$(4,0)$	$(1,1)$

When presented as a one–time game, it is apparent that two rational players will both defect. However, when the game is to be played repeatedly, it has been observed that a cooperation can develop in which both players are made better off. What factors result in this cooperation? Is there an ideal strategy for it?

Much of the thought on this problem is developed more fully in Robert Axelrod's *The Evolution of Cooperation*. In this impressive tome, Axelrod develops the concept of a game environment, in which the payoffs of games signify gains and losses in energy that impact survival within the environment. Successful strategies in the IPD—he claims—have the characteristics of

1. being nice (i.e. never being the first to defect)
2. punishing transgressions firmly (responding to defections)
3. being willing to forgive when the other player stops defecting
4. looking at future payoffs through the lens of an appropriate discount factor (weighting them more than zero but less than their nominal value) [**?**]

Students can learn much about the world from this vein of inquiry, as the social situations modeled are ever present. Most recently in my home town of San Marcos, for example, repairs were begun on a broken dam after three years of deliberation between city and state governments about who would actually pay for it; the problem was that neither entity wanted to commit funds in a situation when the other might not commit funds. This is a classic example of the free rider problem, which is actually equivalent to the Prisoner's Dilemma.

Fair Division of Discrete Lots among a Group: Given a group of people $p_1, p_2, ..., p_k$ and a lot of goods $g_1, g_2, ..., g_k$, what fair methods exist that can help everyone to get what he or she perceives as a "fair" portion of the goods?

In order to create the most equitable division, all players first determine their values of the items in the lot according to a certain system (one may think of these values as being expressed in dollars). Then, an auction is held in which each item is allocated to a high bidder. Finally, through a system of swaps, all players can wind up with what they will perceive as a fair share of the total heap.

Fair division theory will benefit students primarily in two exciting ways. First, it will give them a mathematical tool to use in the most basic interactions within society. Families can divide chores using these methods; groups can divide tasks or resources. The other important concept that is developed within these sorts of problems is the importance of a scale; fair division rests on the ability of those involved to accurately represent the values they place on objects in terms of a metric (usually dollars). This concept is one that junior high students are unlikely to have seen formally; however, the goal–oriented thinking that can be developed through it will help their understanding of the world around them for the rest of their lives.

Kenneth Arrow's Social Choice Theorem: If a group of rational people are to rank a series of alternatives $m_1, m_2, ..., m_k$, why is there no method for doing so that guarantees:

1. Monotonicity (i.e. if several people switch their votes to alternative m_j, then m_j cannot go from winning to losing), and
2. Independence of Irrelevant Alternatives (i.e. if m_k leaves the race, this cannot switch the rankings between m_j and m_i), and
3. Majority Guarantee (i.e. if more than half ot the voters have m_j as their first preference, then m_j cannot lose the election), and
4. Condorcet Dominance (i.e. if m_j would win when pitted one on one against any other alternative, then m_j will win the election).

Unfortunately, we can get any three of these, but we can never get all four. While the proof of this theorem is well beyond the scope of a junior high course, the result is certainly not for the reasons below.

In addition to exposing students to a Nobel-Prize winning result, there are several interesting benefits of the addition of Social Choice Theory to this course.

Students are able to critically evaluate group decision-making for purposes of public policy–making and community activities in which they may be involved.

Also, strange events in our culture can actually be explained using these theories. The two most interesting examples are the Minnesota Governor Election and the 2002 Winter Olympic Women's Figure Skating Championships. In the former, Jesse Ventura, a third party candidate, took advantage of a split vote between the two principals to carry the election with a meager 38% of the vote; this happened despite the fact that—according to polls–60% of the voters would have preferred either mainstream candidate to him. In the latter, audiences were wowed when Irina Slutskaya's performance switched the rankings between Sarah Hughes and Michelle Kwan. As bizarre as both of these events seem, however, Kenneth Arrow has proven them to be in line with the strange events that are possible in any non-monarchic method for social choice.

6 The Future

In the future, several steps need to be made in order to make the Passty Course ready to present; in addition, there are some exciting innovations about which I have learned in the past few months that would truly benefit the course and the students who learn from it.

6.1 Adding the Missing Ingredients

According to Jerome S. Bruner at Harvard University, the following four factors need to be controlled:

1. Experiences of the student must instill a tendency toward learning
2. Knowledge must be structured in a way in which the child can grasp it
3. Sequence of the knowledge must be effective
4. Appropriate accountability must be maintained [**?**]

Mechanisms for doing this within the Passty Course are discussed below.

6.1.1 The Most Important Ingredient

The most important ingredient that need be added to the Passty course is a charismatic instructor. The students will require someone with the tools necessary to excite them about mathematics; the point person in this situation, then, is clearly the instructor. No matter how intricately the lessons are prepared, a human being must inject life into them in order for the students to carry away something of value.

In addition, an instructor is necessary for responding to the the student progress in the course. It would be naïve to expect the students to learn without some form of accountability for that learning. The instructor would play the key role in determining the appropriate method for the students to respond to their learning through the course, be it exams, projects, or an even more exciting method discussed below.

6.1.2 Reversing the Paper Flow

The paper, however, is not meant to flow in only one direction. The diversity of learning styles among gifted students requires that some lessons have handouts. In addition, laboratory activies, such as the Iterated Prisoner's dilemma tournament will require laboratory sheets in order to guide the students throught the lesson; althought preparation of these materials will be a significant responsibility borne by the instructor, one exciting method for creating these is discussed below.

6.1.3 Altering the Format

The Dartmouth Chance Course—perhaps the most ambitious course of this kind—is run in a seminar format. Students are expected to listen to lectures by outside experts, produce newspaper clippings that use (or misuse) mathematics, and participate in lengthy class discussions on problem-solving strategies and methods. The catalyst for developing such mature skills in junior–high level students can be none other than a successful instructor who is able to carefully control communication and learning in all available media.

6.2 Exciting Teaching Innovations to be Added

Four exciting innovations that may have a place in the Passty Course are discussed below. These are: Journaling, non-linear lesson plans, Preview exercises, and interactive handouts.

6.2.1 Journaling

One new medium for student–produced work that has been successfully employed in the chance course has been the journal. In *How to Think Like Leonardo da Vinci*, Michael Gelb argues that good thinking can only be done on paper [**?**] In order to encourage students to truly commit to mathematical thoughts, then, they must write down as many things as they can when confronted by a problem. This has several advantages:

1. dead ends in the working of a problem are carefully preserved so that they may be avoided forever after more
2. successful ideas may be preserved in one place for later consultations
3. new ideas for projects and research may always be found out of places in the notebook; rather than having to search for ideas to explore, the students will have to search for time to explore every idea

4. inspirations about an unrelated subject may be preserved immediately when written in the notebook

Opponents of mathematical journaling will argue that the spiral notebook journal is simply too linear; that there will be no easy way in which to group together ideas concerning similar subjects or to separate out the noise. However, this fits the goals of a non–traditional math course perfectly. Journaling will help students see the linkages between seemingly unrelated problems that are placed right next to each other; this is helpful because this is the way that our brain actually operates: by forming mysterious linkages between thoughts more quickly than we can imagine.

It is very obvious that mathematical journaling will help students learn to find similarities between ideas despite differences between them, and to synthesize new concepts by taking old concepts and putting them together in new ways, two essential abilities of intelligence [**?**].

6.2.2 Non-Linear Lesson Plans

While the packaged nature of the lesson plans that make up the Passty course might seem to encourage them to be run through in a linear fashion, in which each is used to completion before the next, this would probably not be the most interesting to the students or to the professor. In fact, the pre-requisite information is listed on each lesson so as to enable them to be followed out of order.

Dr. Kenneth Nelson suggested that the lessons be broken up into mini–chunks; fortunately, fewer than $\frac{1}{2}$ of the lesson plans are meant to be in the purely constructive "statistical tools" section of the course. What is possible then is for an instructor to schedule just a few minutes each day to focus on teaching one or two new theoretical tools before spending the remainder of the time focusing on highly applied (and highly interesting) questions. This fits in well with the eclectic mix of topics, and is more likely to give the students good project ideas early on in the course.

One danger point of this characteristic is that it will be difficult to ensure that students receive enough drill practice in the techniques before they see the interesting problems. This can be solved by a watchful instructor with a keen eye for accountability. Or, another solution is a careful monitoring of the pre-requisites through our third method:

6.2.3 Preparation Exercises

One of the tiring stigmas associated with drill problem practice is that it follows a lesson. In this sense, then, students can feel as though–rather than an opportunity to learn–it is actually a chore. They may wonder when they will use the material, what the purpose is, etc. One creative solution to this is to assign exercises to be due before the lesson rather than after.

In fact, H. Louise Amick has written an entire Pre-Calculus textbook with this theme, in which the exercises are actually preparatory exercises rather than review exercises. This will turn the working of the problems from a chore into a beautiful anticipatory activity. The purpose, before questioned, is now built in: the exercises must be worked in order to earn the right to learn something. Rather than constricting the direction of learning after the lesson, they build a solid runway from which knowledge can take off and soar.

In order to do this properly, the instructor must have careful control of the pre-requisites for each lesson. Then, the exercises will be designed to test the pre-requisite knowledge required to understand the lesson; if students have any questions about mechanics of math used in the lesson, the answer will already be right in front of them in their journals. Amick found an incredible difference in the tone of her pre-calculus classes when she went to this system, and I suspect strongly that it can be the greatest asset of the Passty course.

6.2.4 Interactive Handouts

Some students learn visually; some learn aurally. If a lecture without notes is presented, then the visual learners may be at a significant disadvantage. However, if they are given handouts, they will tend to ignore the lecture in order to read the handouts. If some method existed for uniting the visual and the aural, that would create an incredible learning power.

Fortunately, David Brooks, the 1990 World Champion of Public Speaking, teaches just such a method: the interactive handout. Preparing one is easy. Prepare a normal handout. Then, remove one key word from each major idea, leaving a blank that will have enough space for the students to write that word in. This will give the visual learners something to look at, while giving the audio learners something to listen for. People who are trying to fill in the blanks on the handout will have a much more difficult time falling asleep or zoning out, and afterwards they could paste the handout right onto a page in their own journal.

These four innovations need not be unique to the Passty Course. However, its topic and spirit are well–suited to them.

7 Conclusion

Over the course of this statement, we have discussed many different aspects of the Passty Course. We've seen how it meets its dual goals of inspiring children to a lifetime of mathematical thinking while training them in specific examples of social mathematics. We've seen how the personal history of the author contributed substantially to the creation of this course. We've compared specificities of the course to other similar courses that already exist (with the Dartmouth Chance course obviously setting the standard). And we've seen a succinct tour of the material within the course. Finally, we've seen how all of the above can be synthesized by a charistmatic instructor—with many different teaching innovations at his or her fingertips—into a terrifically eclectic multi-week experience for extremely gifted or motivated junior high students.

8 Appendix First – Useful Publications

Peterson, Elisha and Francis Edward Su. Four-Person Envy-Free Chore Division. Manuscript Version, November 2000.

Haake, Claus-Jochen, Matthias G. Raith and Francis Edward Su. Bidding for Envy-Freeness: A Procedural Approach to n-Player Fair-Division Problems. To Appear in Social Choice and Welfare, Feb. 2001.

Taylor, Alan D. Mathematics and Politics: Strategy, Voting, Power, and Proof. New York: Springer-Verlag, 1995.

Tannenbaum, Peter and Robert Arnold. Excursions in Modern Mathematics. New Jersey: Prentice-Hall, 1998.

Exploring Surveys and Information from Samples (book from Dr. Cooper)

Exploring Data (Dr. Cooper Book)

The art and Techniques of Simulation (Dr. Cooper Book)

9 Appendix Second – Websites and Other Resources

`http://www.jump.net/~jnhtx/ec/ec.html` electoral college java applet

`http://www.maa.org/mathland/mathland_5_13.html` Brian Dawsons Choosing sides, which was found from link at

`http://cwx.prenhall.com/bookbind/pubbooks/tannenbaum/`

`http://www.canadianlessonplans.com/aclp/math/index.html` Canadian Standards for education (alternatively, see

`http://www.canadianlessonplans.com/aclp/math/level_7/scos7g.html`

Paul Uhligs lecture on Fair Division, given to the Trinity University Math Majors' Seminar given on January 31, 2002

`http://www.pbs.org/teachersource/math/high_statistics.shtm` PBS teacher source

`http://www.ucs.mun.ca/~mathed/Stats/intro18.htm` Data analysis and discrete math

`http://www.ucs.mun.ca/~mathed/Stats/intro10.htm` Probability: The Study of Chance

`http://www.awesomelibrary.org/` So many interesting educational references

`http://www.maa.org/mathland/mathtrek_11_23_98.html` birthday problem

`http://www.mste.uiuc.edu/activity/rocket/default.html` Great Java Applet

`http://explorer.scrtec.org/ Lesson Plans` (some even for statistics and probability)

`http://askeric.org/Virtual/Lessons/` Ask Eric Clearinghouse

`http://problems.math.umr.edu/index.htm` Math Problem Search Engine

`http://mathforum.org/dr.math/faq/faq.monty.hall.html` MHD http://mathforum.org/isaac/problems/prob1.html Problem of Points

http://mathforum.org/library/topics/probability/ Good problems in Probability

http://mathforum.org/epigone/alt.math.undergrad/ Math Forum

http://www.maa.org/t_and_l/exchange/ite1/ite1.html Preview Lesson

http://www.dartmouth.edu/~chance/ Dartmouth Chance Course Database

http://www.enc.org/ Another Clearinghouse

http://www.nctm.org Standards of the National Council of Teachers of Mathematics

Unpublished "Oligopoly and Game Theory" lesson plan for Economists in the Schools Program by Keith Xavier as well as several lesson plans for the Probability and Game Theory course at the Johns Hopkins CTY Summer Program

References

[1] R. M. Axelrod, *The Evolution of Cooperation*, Basic Books, New York, 1984.

[2] D. Buckey, *Trends in school mathematics*, Great Lakes Collaborative, Jan. 24 (1996).

[3] M. Gardner, *The Colossal Book of Mathematics*, W. W. Norton and Company, New York, 2001.

[4] M. J. Gelb, *How to Think like Leonardo da Vinci*, Delacorte Press, New York, 1998.

[5] D. R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, 1979.

[6] D. Huff, *How to Lie with Statistics*, W. W. Norton and Company, New York, 1954.