Optimal Pricing Strategy for Second Degree Price Discrimination

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Abstract

Second Degree price discrimination is a coupon strategy that allows all consumers access to the coupon. Purchases are made by consumer self selection based on utility, a function of shelf price, coupon face value, and hassle cost. This work builds on a previous study and develops a graphical analysis and model based on similar assumptions. We find an optimal shelf price and coupon value to segment a population based on maximizing a firm's profit.

1 Introduction

This article addresses the questions "Why would a firm simultaneously lower the retail price and offer a coupon?" and "When and why is this strategy optimal?" We combine Mathematics and Marketing to discover the combination of both disciplines in comparing our work with that from an artical in the November 2004 issue of the *Journal of Marketing Research* [1]. We develop a model based on similar assumptions that has a graphical representation. This newfound knowledge is applied to better understand firm and consumer behavior.

Over the past few decades, coupons have grown drastically as a promotional tool. In the 1980's, this growth was at an average rate in excess of 11%. In 1993 more than 3,000 coupons per household were distributed [2]. Companies frequently offer coupons for consumer trials, as incentives to purchase a product, to encourage brand switching, to create brand loyalty, to gather information about price sensitivity, and to increase sales for a short duration.

A fundamental assumption is that a company is not trying to gain the business of every consumer in the market. One responsibility of a marketing manager is to segment the population into different types of consumers and then find the best way to appeal to these segments to increase profits. In these segments, there are types of people that will not buy your product no matter how it is priced. For this reason, it is not profitable to price your products in a manner that appeals to everyone.

This is where price segmentation comes into play. There are many different strategies for price segmentation. For example, a skimming strategy is used with an introductory product in which a firm prices the product extremely high to take advantage of the consumers who purchase at any price in order to be leaders in the market. The firm then gradually decreases the price until one is found that appeals to a majority of the market. In this paper, we are concerned with *Second Degree price discrimination*, or offering a coupon so it is available to all consumers. For example, a firm may place a coupon in a newspaper to allow all consumers access to this coupon. Similar couponing methods are *Third Degree price discrimination*, which refers to the targeting of consumer segments, and *First Degree price discrimination*, also known as perfect price discrimination, which refers to the targeting of individual consumers.

Second Degree price discrimination aims to segment the population by valuation of the product and by price sensitivity. Thus, purchases are made based on consumer self selection rather than firm targeting. This has 3 main effects on profit. The negative effect of lost profit from consumers normally willing to purchase at the regular price but who choose to use a coupon when one is offered is called the *trading down effect*. Ideally the trading down effect is zero, but this is nearly impossible in Second Degree price discrimination. The positive effect on profit from consumers who purchase the product with a coupon that would not have purchased at the regular price is called the *new customer effect*. The change in profit resulting from changes in the regular price when a coupon is offered is called the *regular price effect* and is either positive or negative. The model presented later attempts to offset the positive and negative effects by setting price and coupon value in such a way as to maximize profit.

Because we are using Second Degree price discrimination, the hassle cost of using a coupon plays a major role in the decision making. We now introduce the idea of utility. Utility is a term used to describe benefit. For firms, utility is simply profit. For consumers, utility is the gain from purchasing a product and is a function of price, hassle cost, coupon value, and intrinsic social value. Throughout, *utility* is used to refer to consumer utility, and *profit* is used to refer to firm utility. Our model of utility is described in detail later.

We investigate optimal strategies for setting price and coupon value in order to maximize profit. This increases overall social welfare, as both the firm increases its profits and most consumers pay less money. The goal of this paper is to lay a foundation of common assumptions and provide a single uniform model that is able to describe complex situations with a graphical representation. This helps us understand the relationships between coupon face value, shelf price, and hassle cost.

2 Model Assumptions

The model considers a single firm selling to a unit mass of customers of type θ, where θ is uniformly distributed on the interval [0,1]. The firm sells only one type of product at price $p \geq 0$ with a marginal cost of $w \geq 0$. Utility is modeled by a piecewise linear function of q, where $q \geq 0$ is data driven and depicts the valuation increase across the θ interval. That is, q is the rate at which utility increases over the population. Furthermore, $\alpha \geq 0$ is the minimum value, or intrinsic value, the product has to society. Hassle cost, $H(\theta) \geq 0$, is positive for only a fraction β of consumers of type $\theta \geq X$, where X delineates where consumers begin to have a hassle cost that effects utility. Hassle cost is discussed later in detail. The value of X is determined by the data of the specific firm, product, and most importantly consumers. The utility function, $U(\theta)$, for each consumer of type θ is determined by weather or not they consider using a coupon with a face value $c \geq 0$. It is important to note that we assume consumers have certain characteristics and purchasing patterns in the market, and these characteristics are what determines α, β, X, q, w, and h. That is, these parameters are data driven. The consumer utility function is

$$U(\theta) = \begin{cases} U_C(\theta), & 0 \leq \theta \leq (1+\beta)X \\ U_{NC}(\theta), & (1+\beta)X < \theta \leq 1, \end{cases}$$

where

$$U_C(\theta) = U_{NC}(\theta) - H + c \tag{1}$$

and

$$U_{NC}(\theta) = \alpha + \theta q - p. \tag{2}$$

The utility functions, $U_C(\theta)$ and $U_{NC}(\theta)$, are restricted to their domains, as shown in Figure (1). We call $U_{NC}(\theta)$ utility with no consideration of a coupon and $U_C(\theta)$ utility with consideration of a coupon. Consumers of type θ are not allowed to choose which function they measure their utility with, but at the same time, it is their consumption patterns upon which utility is based. For simplification, we assume that income increases as θ increases. That is, consumers of type θ close to zero have a low income and consumers of type θ close to 1 have a high income, and it is in this way that the utility domains are chosen. In other words, the wealthier consumers do not consider using a coupon while the poorer consumers do consider using a coupon. The middle class consumers have a positive hassle cost and must determine if the coupon value outweighs their hassle cost.

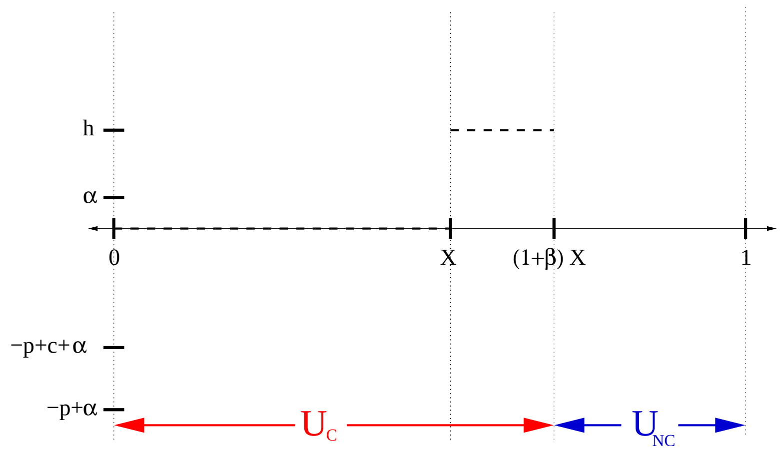

Figure 1: Utility Function Domains

If $U(\theta) \geq 0$, then we say that consumer θ purchases the product. A consumer foregoes purchasing if their utility is negative. Notice that

$$U_{NC}(\theta) \geq 0 \Leftrightarrow \theta \geq \frac{(p-\alpha)}{q}, \tag{3}$$

and

$$\begin{aligned} U_C(\theta) \geq 0 \Leftrightarrow \theta &\geq \left[\frac{(p-\alpha)}{q} - \frac{(c-H)}{q}\right] \\ &= \frac{p-c-\alpha+H}{q}. \end{aligned} \tag{4}$$

Hassle cost is the consumer's disutility from using a coupon and is a time consideration. For example, the hassle of keeping track of the coupon, organization costs, or a variety of other "costs" that influence a consumer's desire to use a coupon. Hassle cost is defined by

$$H(\theta) = \begin{cases} 0, & 0 \leq \theta \leq X \\ h, & X \leq \theta \leq (1+\beta X), \end{cases}$$

where $h > 0$. Again, notice that hassle cost is only a factor in utility for a fraction β of consumers of type $\theta \geq X$. Note that consumers of type $\theta \in ((1+\beta X), 1]$ do not consider the use of coupons because they are too wealthy, thus, they have no applicable hassle cost. Hassle cost is depicted in the figures as the dashed piecewise constant function.

Since utility is the determining factor of purchases, we are concerned with where $U(\theta) \geq 0$. Specifically, we determine exactly where $U(\theta) = 0$, say at $\hat{\theta}$, and consumers of type $\theta \geq \hat{\theta}$ purchase for all θ within the respective utility domain. Although $U(\theta)$ is piecewise linear, it is not continuous, and thus utility can alternate between positive and negative along the θ axis, as seen in Figure (2). Note that $U(\theta) = 0$ can have 3 solutions.

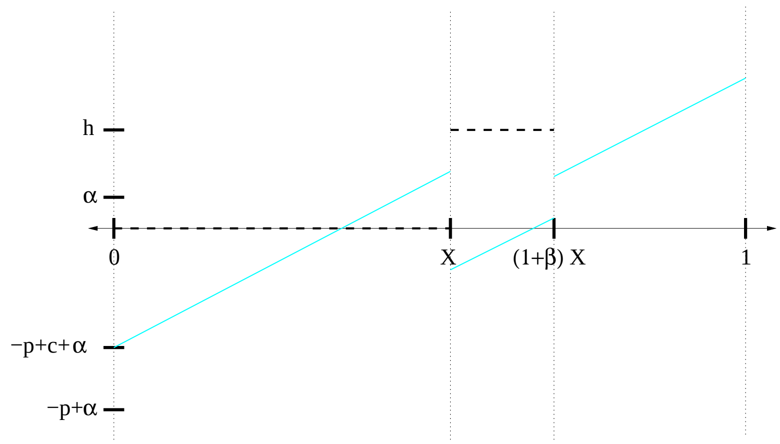

Figure 2: Graphical representation for $U(\theta)$

3 A Model of Profit, Segmentation, and Utility with a Graphical Representation and Analysis

We now develop a model that determines an optimal price and coupon value that segments the population to maximize profit. Profit, π, is obtained where utility is non-negative, or where consumers purchase the product. Remember that profit is calculated by price, p, less cost, w, less any losses from a coupon, c, multiplied by the quantity sold. The model calculates quantity sold as the number of consumers with a non-negative utility. That is, quantity sold is the sum of the distance of all $\theta \in [0, 1]$ where utility is non-negative, or where $U(\theta) \geq 0$. Let $V = (p, c, a_2, s_2, s_3, s_4)$. The model is as follows:

$$\max \pi(V) = a_2(p - c - w)s_2 + (p - c - w)s_3 + (p - w)[1 - ((1 + \beta)X + s_4)]$$

Subject to:

$$\frac{p - c - \alpha}{q} = X + a_1 s_1 - a_2 s_2, \tag{5}$$

$$a_1, a_2 \in \{0, 1\}, \tag{6}$$

$$\sum_i a_i = 1, \tag{7}$$

$$X \leq \frac{p - c - \alpha + h}{q} + s_3 \leq (1 + \beta)X \tag{8}$$

$$\frac{p - \alpha}{q} - s_4 = (1 + \beta)X \tag{9}$$

$$0 \leq s_1 \leq \beta X, \tag{10}$$

$$0 \le s_2 \le X, \tag{11}$$
$$0 \le s_3 \le \beta X, \tag{12}$$
$$0 \le s_4 \le (1 - (1+\beta)X), \tag{13}$$
$$p, c \ge 0. \tag{14}$$

To understand the model, we first explain the constraints. Consider constraints (5) through (7). From the domain of $U(\theta)$ and $H(\theta)$, we know that $U(\theta) = 0$ when $\theta = (p-c-\alpha)/q$ for some $\theta \in [0, (1+\beta)X]$. Note that $H(\theta) = 0$ for $\theta \in [1, X)$ and remember that X is the point determined by the data of the population where hassle cost is positive for all $\theta \in [X, (1+\beta)X]$. Thus we set $(p-c-\alpha)/q = X + a_1 s_1 - a_2 s_2$, where a_1 and a_2 are binary variables. Notice that s_1 is the distance from X to $(p-c-\alpha)/q$ when $(p-c-\alpha)/q > X$, and s_2 is the distance from $(p-c-\alpha)/q$ to X when $(p-c-\alpha)/q < X$. Constraint (7) allows only one of the a_i's to be 1 so that either s_1 or s_2 is removed from the profit calculation. We let $0 \le s_1 \le \beta X$ and $0 \le s_2 \le X$ which guarantees $0 \le (p-c-\alpha)/q \le (1+\beta)X$, and thus utility becomes non-negative within our population and is restricted to the specified utility domain.

If $(p-c-\alpha)/q < X$, then profit is increased by $(p-c-w)s_2$. If $(p-c-\alpha)/q = X$, then $s_1 = s_2 = 0$ and there is no increase in profit. If $(p-c-\alpha)/q > X$, then profit is not effected by s_1. Although we allow for $X < (p-c-\alpha)/q \le (1+\beta)X$, if the utility becomes nonnegative for some $\theta \in (X, (1+\beta)X]$, then we use s_3 for profit calculation within this domain and s_1 is simply used as a reference for utility. That is, if $X < (p-c-\alpha)/q$, then hassle cost drops the utility function down and allows contstraint (8) to choose an s_3. Figures (3) and (4) depict a graph of $U(\theta)$ that shows the existence of s_1 and s_3. However, $U(\theta) < 0$ over $[0, X)$, so there is no s_1 and therefore no additional profit is made by the firm from this segment.

Figure (3) represents an infeasible graph of $U(\theta)$, but we see a corresponding feasible graph of $U(\theta)$ in Figure (4). That is, our model does now allow Figure (3) because $U(\theta) \ne 0$ for $(1+\beta)X \le \theta \le 1$. Thus, the model forces p to decrease. We then obtain an intersection depicted by the feasible graph of $U(\theta)$ in Figure (4). This is clarified with the understanding of the remaining constraints.

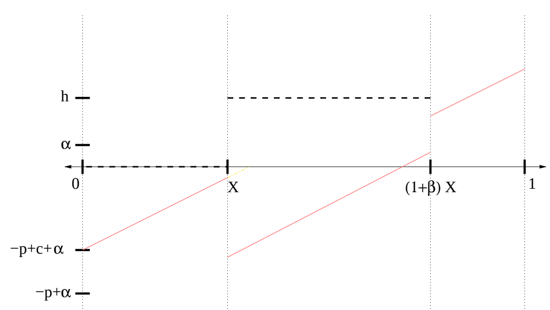

Figure 3: Infeasible graph of $U(\theta)$

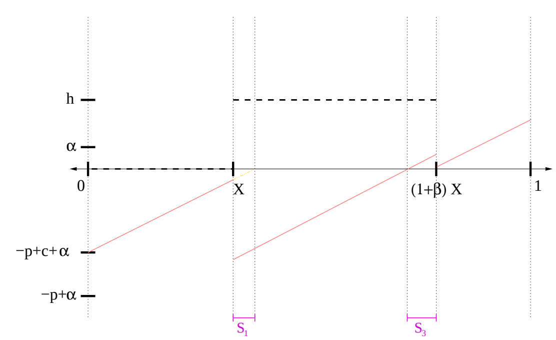

Figure 4: Corresponding feasible graph of $U(\theta)$, with $s_1, s_3 > 0$

Again, constraint (8) is used to determine utility on the middle segment. On this interval,

$H(\theta) = h$ and s_3 is the distance between $(p - c - \alpha + h)/q$ and $(1 + \beta)X$, or where $U(\theta) \geq 0$ for $\theta \in [X, (1 + \beta)X]$. We see that the objective function drives s_3 to be as large as possible and that profit for this segment is increased by $(p - c - w)s_3$.

Figure (5) depicts another infeasible graph of $U(\theta)$ while Figure (6) depicts the corresponding feasible graph with positive s_2 and s_3. Again notice how the model forces the utility function to change in Figure (6). Constraint (12) bounds s_3 so that we keep $(p - c - \alpha + h)/q$ in the correct domain.

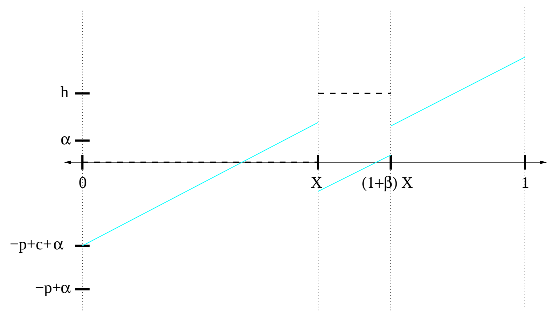

Figure 5: Infeasible graph of $U(\theta)$

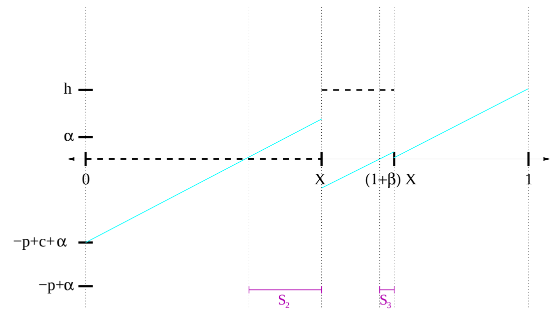

Figure 6: Corresponding feasible graph of $U(\theta)$, with $s_2, s_3 > 0$

Due to the domain divisions of $U(\theta)$ and constraint (9), we see that $(p - \alpha)/q = 0$ for some $\theta \in [(1 + \beta)X, 1]$. We let s_4 be the distance between $(1 + \beta)X$ and $(p - \alpha)/q$, or where $U(\theta) < 0$. Because of this, we see in the objective function that the distance between $(1 + \beta)X$ and 1 where $U(\theta) \geq 0$ is $[1 - ((1 + \beta)X + s_4)]$. On this domain coupons are not considered, and thus profit is not effected by c. Therefore profit increases by $(p - w)[1 - ((1 + \beta)X + s_4)]$. Note that s_4 is bounded by constraint (13). Figure (7) represents yet another infeasible graph of $U(\theta)$ and Figure (8) shows the corresponding feasible graph with $s_4 > 0$.

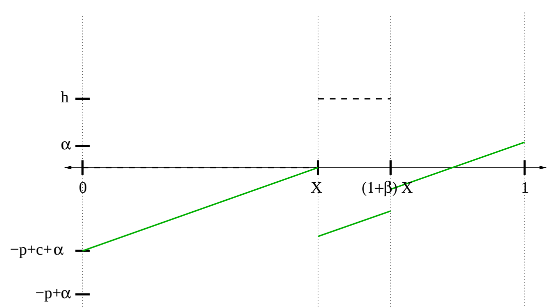

Figure 7: Infeasible graph of $U(\theta)$

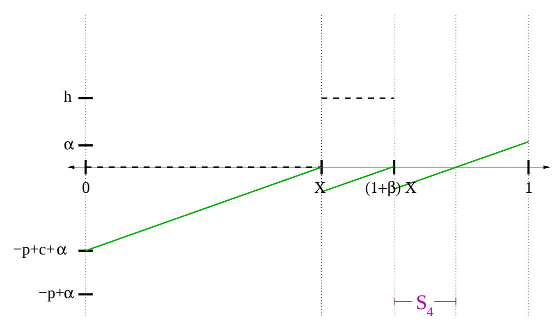

Figure 8: Corresponding feasible graph of $U(\theta)$, with $s_4 > 0$

Profit is higher when s_2 and s_3 are larger, and when s_4 is smaller. However, the other variables in the model greatly effect wheather the objective will attempt to minimize or maximize any of the s_i's.

4 Bounds on p and c

We now make some observations on p and c and see that our model constraints provide bounds for both p and c.

Theorem 4.1 *The model constraints provide a lower bound for c and both an upper and a lower bound for p. In particular,*

$$\begin{array}{rcccl} h & \leq & c, & & \textit{and} \\ (1+\beta)Xq+\alpha & \leq & p & \leq & q+\alpha. \end{array}$$

Proof: We first show $h \leq c$. From constraints (8) and (9) we have that

$$X \leq \frac{p-c-\alpha+h}{q} + s_3 \leq \frac{p-\alpha}{q} - s_4, \tag{15}$$

which implies

$$qX - p + \alpha \leq -c + h + qs_3 \leq -qs_4. \tag{16}$$

Since $s_4 \geq 0$, we have that $-qs_4 \leq 0$. Therefore,

$$-c + h + qs_3 \leq 0. \tag{17}$$

Since $qs_3 \geq 0$, we conclude that

$$h \leq c. \tag{18}$$

We now show the bounds on p. From constraint (14), $p \geq 0$. However, we can find a greater lower bound by using constraint (9). We see that

$$\begin{array}{rcl} \dfrac{p-\alpha}{q} - s_4 & = & (1+\beta)X \\ \Rightarrow p & = & (1+\beta)Xq + \alpha + s_4. \end{array} \tag{19}$$

Since $s_4 \geq 0$, we have that

$$p \geq (1+\beta)Xq + \alpha > 0. \tag{20}$$

Thus, we have shown a lower bound on p.

We use equation (19) and notice from constraint (13) that $s_4 \leq (1-(1+\beta)X)$. Thus, we substitute into equation (19) to obtain

$$\begin{array}{rcl} p & \leq & (1+\beta)Xq + q(1-(1+\beta)X) + \alpha \\ & = & q + \alpha. \end{array}$$

Therefore, we have that $(1+\beta)Xq + \alpha \leq p \leq q + \alpha$. ■

Parameter or Characteristic	Assumed Value for Example
X	1/2
β	1/3
q	8
h	2
α	1
w	2

Table 1: Parameters for Example

p	c	π
$9	1	$0
$19/3	1	$1,444
$7	1	$1,250
$6	1	$1,333
$9	2	$0
$19/3	2	$1,638
$7	2	$1,250
$6	2	$1,833
$9	3	$0
$19/3	3	$1,889
$7	3	$1,583
$6	3	$1,916

Table 2: Values for p, c, and π

5 A Numerical Example

The following example contains different scenarios for values of p and c and demonstrates the difficulty in determining an optimal p and c. We first make educated assumptions and pick values for our population parameters. Table (1) summarizes these parameters.

Selecting values for p and c, we see their effect on profit, π, as shown in Table (2). Note that the bounds on p give us an upper bound of \$9 and a lower bound of \$19/3, or \$6.33. From the table, we have that when p is its upper bound, the firm gains zero profit. Thus, it is not optimal for p to be its upper bound. We also notice that the lower bound on p may be optimal. Note that the first four cases with $c = 1$ are not allowed by our model. Furthermore, the three cases where $p = \$6$ are also not allowed by our model, as the lower bound on p is \$6.33. These cases are simply used as a reference to see the effects of different p and c values on profit. For profit calculations we assume that θ is representative of 1,000 people in our population. Thus we multiply profit by 1,000 to gain a more realistic dollar value.

From this example we see that the highest profit obtained is when $p = \$6$ and $c = 3$.

Literature Model		Our Model
utility function domains dependent on p and c		fixed domains for utility
2 cases		one model
p unbounded		p bounded
forces $c \leq h$		ensures $c \geq h$
$c = h$ is optimal		claim $c > h$ is optimal

Table 3: Comparison of models

However, this p value is not attainable in our model. The highest profit allowed by our model is when $p = \$19/3$, the lower bound on p, and when $c = 3 > h$. This leads us to Conjecture (5.1).

Conjecture 5.1 *If (c^*,p^*) is optimal, then $h < c^* < (p^*\text{-}w)$. Also, there exists an optimal (c^*,p^*) such that $p^* = (1+\beta)Xq + \alpha$.*

In comparing our model with the one in an article from the *Journal of Marketing Research* [1], we discover some interesting differences. Table (3) summarizes our comparison. Our model restricts the domain of the utility function while the literature allows the utility of consumer θ to change depending on the firms desicions. That is, the literature's model allows consumers to measure their utility differently depending on θ, p, and c. Because of this, the literature is forced to divide the model into two cases, $X < (p-\alpha)/(q)$ and $X \geq (p-\alpha)/(q)$. We develop a uniform model that accounts for all possiblilities through the use of binary variables and a new utility function.

Because of the fixed domains of utility and the constraints that force $U(\theta) \geq 0$ for each domain of $U(\theta)$, we find that our model places bounds on p. At first glance this seems to restrict the amount of possible profit, but upon further investigation, we see that the model also insures $c \geq h$. Since c is allowed to increase beyond h, it essentially reduces the price consumers pay and thus increases their utility so that we obtain an equivalent or greater number of consumers with a non negative utility than the literature. The literature does not place bounds on p, but does however add an explicit constraint in their model that forces $c \leq h$.

From a marketing standpoint there is always some kind of relationship between hassle cost and coupon value. Most marketing managers would argue that allowing coupon value to exceed hassle cost is detrimental to profit as it would not accurately segment the population. The firm is essentially allowing the market a discounted price and not gaining sales through price sensitivity segmentation. However, we have used similar assumptions in developing our model and still find an optimal p and c that maximizes profit given these assumptions. We assume that h is independent of c, but the literature assumes h is dependent on c. The literature also forces $c \leq h$ to agree with the insight of the marketing assumptions. We find that this constraint is not needed. In fact, using similar assumptions to develop our model, we find that the opposite is true.

In our example, $c > h$ provides a higher profit than $c = h$. We find this to be true because increasing c essentially reduces p to be below the lower bound for all consumers that consider a coupon. Thus again we are lead to believe that conjecture (5.1) is true. While we claim $c > h$ is optimal for our model, the literature finds that $c = h$ is optimal for their model. Note that this is their upper bound on c.

In this paper we combine two cases in [1] to develop an alternative model based on similar assumptions that bound p and c. We find it interesting that our model ensures $c \geq h$ while the literature forces $c \leq h$. However, this is not suprising. Because we ensure $c \geq h$, we still have a feasible optimal solution for maximizing profit. We claim that $c > h$ and the lower bound on p is optimal. However, because we assume a piecewise utility function with fixed domains $U(\theta)$, we are over simplifying the population. Therefore, it may not be realistic to apply our model to an empiricle coupon offering.

6 Conclusions

This paper is beneficial to both firms and consumers. For consumers, the understanding of this paper is most beneficial in understanding their purchasing behaviors. For firms, when offering a coupon in a Second Degree price discrimination situation, the firm can find an optimal price and coupon value that maximizes profit. Furthermore, the firm gains an optimal segmentation of the population. For these reasons, a whole field of study in marketing is devoted to consumer behavior, which includes the study of hassle cost and purchasing patterns.

In conclusion, this newfound knowledge can be applied to better understand firm and consumer behavior.

References

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[2] Greg Shaffer and Z. John Zhang. Competitive coupon targeting. *Marketing Science*, 14:395–396, 1995.