Reviewing Reformed Calculus

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"The calculus is one of man's great intellectual achievements; no educated man or woman should be wholly ignorant of its elements." Tony Ralston

Abstract

The movement to reform the Calculus curriculum began in the late 1980's in an attempt to increase both passing rates and a general interest in mathematics. The motivations behind the curriculum reform have led to many changes in the way that reformed Calculus is taught and how students interact with the material. Studies have been conducted at the university level to evaluate the effectiveness of reformed Calculus in comparison to traditional Calculus. There are many difficulties that researchers have encountered in conducting these studies, which as a result, does not provide a clear picture whether reformed Calculus is having the desired effect. In addition, it is difficult to compare the results from different universities since there is no standard method of evaluation. An experimental design structure is suggested that would provide a standard evaluation method that would ideally eliminate the difficulties and bias to allow for data comparison across universities.

Keywords: Calculus, Reform

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1 Introduction

Calculus has been a staple of a college education for well over one hundred years. Over the course of those years the method of instruction has remained generally the same. For decades the traditional "plug and chug" method has been the top choice for Calculus education. The trademarks of this traditional instruction are rooted in the rigorous development of the classic contributions of Newton, Leibniz, Reimann, and Euler. Rote memorization of formulas, algebraic manipulation and a significant amount of time working problems with pencil and paper have long been associated with learning the art of Calculus. Despite the long tradition of rigorous instruction, a movement began that called for an update of the Calculus curriculum. During the 1980's, some mathematicians and educators began to argue that traditional Calculus instruction was not the most effective means of teaching. With very few students taking more than the required math course for graduation, these educational "reformers" argued that Calculus education needed to be restructured. These educators wanted a new Calculus that would make greater use of technology and emphasize the practical applications of Calculus. As the movement to reform Calculus curriculum began, mathematicians and educators in favor of maintaining the traditional Calculus instruction immediately began voicing their opposition. Since Calculus is one of the major components of a mathematics education, the debate over instruction struck the heart of mathematics. The "math wars", as the reformed vs. traditional debate is sometimes referred to, have been marked by heated debates at all levels of mathematics. Some college mathematics departments have even split into factions and moved into separate buildings over the issue of reformed Calculus. Brad Osgood, a reform professor at Stanford notes that reformed Calculus is "a battle for the soul of the profession" [14]. This paper attempts to show the differences between traditional and reformed curriculum. In addition a sampling of studies on reformed Calculus are evaluated both for their results and for their implementation. The difficulties of designing and performing such studies is discussed, and a possible new experimental design for evaluating reformed Calculus is presented.

2 Traditional Calculus

Until the recent past, there was not much debate on how Calculus should be taught. The method of instruction had remained unchanged for decades. Over the years traditional Calculus has developed several distinguishing characteristics. One of the most notable is perhaps the fact that there is essentially only one curriculum. There may be minor differences in the order in which topics are covered, but for the most part, traditional Calculus is traditional Calculus no matter where or by whom it is taught.

As with the continuity of curriculum, traditional Calculus consistently employs the pencil and paper approach to learning. The old adage 'practice makes perfect' is an adequate descriptor of traditional Calculus. Through drilling specific calculations and topics, the student learns and masters the concept. For homework, students usually are assigned a large set of problems on the same topic that have minor variations. This allows the student to practice and commit to memory the concepts that have been covered.

Traditional Calculus curriculum also reflects the rigor of Calculus with technical mathematical language and symbols being used. In addition, theorems and proofs are a part of the coursework, although not to the same extent of upper-level courses. Students may not necessarily be asked to write a proof but they are exposed to various proofs and are expected to use theorems in order solve problems.

Additionally, traditional Calculus is taught in a teacher-centered classroom. In other words, the teacher is the primary source of learning and knowledge. The lessons are presented in a lecture format where the teacher explains new concepts and then provides computational examples of the new concept. In the traditional classroom, the student plays a secondary role. Students simply take notes during the lecture and ask the teacher for clarification when needed. The primary method of study is one in which a student sits with a textbook to read and learn. Generally, this aspect of traditional Calculus reflects the individual pursuit of mathematical knowledge.

3 Reformed Calculus

The very beginnings of mathematics reform started in the 1960's, but the big push for Calculus reform started in earnest in 1989 with the publication of the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for Mathematics Education*. The NCTM published the *Principles and Standards* in response to the apathy of students towards math and the lack of academic success in the mathematics classroom. To combat these negative trends the NCTM outlined five goals for "the processes of problem solving, reasoning and proof, connections, communication, and representation" [1]. Through these goals it was hoped that students would be equipped with the basic skills and understanding that they would need to be successful.

As the *Principles and Standards* inspired the reform of secondary mathematics education, thoughts of reform began to surface in the collegiate mathematics arena, especially with regard to Calculus. College Calculus courses were experiencing some of the same problems as secondary mathematics. Of the roughly 300,000 college students that are annually enrolled in an engineering-based Calculus course, only 140,000 earn a grade of D or higher [6]. Less than half of the students were performing 'well' in their Calculus courses. Armed with statistics such as this, reform-minded professors set out to develop a new curriculum that would help raise the achievement level and stimulate student interest in mathematics.

From the reform movement, numerous curricular designs have been generated. Calculus and *Mathematica*; Calculus, Concepts, Computers and Cooperative Learning (C^4L); and The Calculus Consortium at Harvard (CCH) are a few of the commonly used curriculums. These new curriculums cover the entire spectrum of reform. Some are grounded in traditional techniques but incorporate snippets of reform, while others differ in most aspects from the traditional approach. Despite this vast array, there are some basic elements that are common to all reform curriculums in varying degrees that separate them from the traditional Calculus curriculum.

One of the most noticeable differences of reformed Calculus is the use of graphing calculators and/or computers. The graphing calculator is a critical component in the reform classroom. Many reform classes include a weekly lab session where students meet in a computer lab. The students make use of calculators and math computer programs to investigate new topics and to graphically see what they are working on. Most reform textbooks urge students to read through the text with a calculator in hand to see directly what is discussed in the text. The idea behind the incorporation of calculators and computers is to alleviate the heavy algebraic manipulation that students typically do in a traditional Calculus setting. Reform supporters argue that the removal of manipulation allows students to move beyond the drudgery of computation and start learning the fundamental ideas of

Calculus. They additionally argue that topics are discussed more fully with the use of graphical representations.

A reformed Calculus class differs from a traditional course in methods of instruction. When walking into a reform classroom it is immediately clear that it is indeed a reform classroom. Most noticeably the teacher is no longer the central focus of the classroom experience. The lecture method of instruction, a standard of traditional curriculum, has a lesser place within a reform setting. The teacher still lectures occasionally and is available to answer questions from the students, but there is greater emphasis placed on cooperative learning. Reform students often work in groups to determine solutions or to explore concepts in a laboratory setting. This idea is rooted in the constructivist learning theory. Each student constructs their own meaning as they learn. Students are given the basic tools and from these discover how the pieces fit together to form the concept that they are studying. One of the primary goals of the C^4L curriculum is to "create situations which foster students to make the necessary mental constructions to learn mathematics concepts" [10].

Within the curriculum itself, the reformed method stresses the applications of Calculus. This emphasis hopes to justify the topics of study, which in theory raises interest in the material. In an effort to accomplish this, some of the mathematical rigor is removed from the curriculum. Most reform textbooks are void of a single proof. In the introduction to *Calculus from Graphical, Numerical and Symbolic Point of View*, the authors state that "proving theorems in full generality is less valuable, we think, than helping students understand concretely what theorems say" [9]. As a result of this change, a common question that arises from students new to reformed Calculus is "Where is the math?" [5].

Accompanying this application heavy curriculum is a different method of assessment. Reformed Calculus courses emphasize the use of writing. Projects, reports and lengthy explanations of problem solutions are common place within the reform classroom. In some cases, the students are graded more on the thoroughness and completeness of written explanations as opposed to correctness of answer.

The following is a sample midterm question from a Duke University reformed Calculus test

> Directions: When you have solved the problem, write a complete explanation in essay form; you will be graded primarily on the quality of your explanation. You may use text, project, and lab materials, notes, calculator and anything else you have with you.
>
> Problem: If the population P of the world is measured in billions and time t is measured in years, the historical data suggests that the growth rate of P can be modeled by $dP/dt = 0.005P^2$. Given the fact that the population reached 5 billion about the start of 1987, this growth rate predicts that P becomes infinite about the start of 2027.
>
> We consider now a fanciful scenario in which the world population problem is complicated by a small but steady immigration from outer space: Alien beings that are identical to humans in every way start arriving in 1987 at the rate of 5 million (0.005 billion) per year. Since the immigrants are traveling from a distant solar system (in a state of suspended animation, so they arrive with normal ages in Earth years), there is no way to turn off this steady flow-the travelers for the next several years are already en route.
>
> Thus, from 1987 on, the growth rate for world population has an added component to

> account for immigration. That should mean that P is predicted to become infinite even earlier. How much earlier? [12]

The emphasis on correct explanation rather than correct answer is seen explicitly in the directions for the midterm. The problem also provides an example of the type of application problems that reformed Calculus students are accustomed to working with. This midterm question additionally exhibits one of the flaws that traditional professors are quick to point out. The problem asks students to determine when the population becomes infinite, which is a misuse of the word infinite. The population may become uncontrollable but it will never become infinite. Traditional professors argue that the misuse of mathematical terms, such as infinite, teaches students the wrong meaning of or concept behind the term, which results in misunderstandings in future math work.

More generally, there is a trend in reformed Calculus moving away from individual study and towards a social study of Calculus. The context of learning Calculus is now placed in a more social setting. Students work primarily in groups to gain knowledge both from a textbook and from each other.

4 Textbooks

Another significant element of the reform movement has been the development of reformed Calculus textbooks. As can be imagined with the development of the reformed curriculum, these textbooks have been designed to cater to the above mentioned elements of reform. In the quintessential reform textbook, *Calculus* [7], sometimes simply referred to as "The Harvard Book", the authors layout two of the guiding principles used in writing the book. First, 'The Rule of Three' states that every topic should be presented geometrically, numerically and algebraically. The second principle is 'The Way of Archimedes'. This is essentially a restatement of the constructive learning theory stating that formal definitions and procedures evolve from the investigation of practical problems [7].

In addition to obvious differences in the layout of reform textbooks (significantly more charts and graphs, page long problem descriptions, and lack of proofs) one of the biggest differences lies in what topics are covered. The reformed Calculus textbooks adhere to the idea of "lean and lively" Calculus. This term coined by Ronald Douglas, the father of the reformed Calculus movement, reflects an effort to remove the topics that are least important to the Calculus curriculum. In an effort to accomplish this "lean and lively" text and to allow for deeper student understanding of topics, the reform curriculum has eliminated some of what is claimed to be nonessential. Some reformed Calculus professors feel strongly about the topics that have been removed from the traditional curriculum. Brown, Porta and Uhl, reform professors from the University of Illinois at Urbana-Champaign, comment that

> Missing [from reformed Calculus] are topics like limit at a point, relative max-min, Rolle's theorem, second derivative test, Riemann sum definition of integral, definition of logarithm via integration, shell vs. disk method, many of the mindless applications of the integral like surface area of solids of revolution, Simpson's rule, serious treatment of L'Hopitals rules, formula for the error term in Taylor Series. These are all filler topics in one-variable calculus. They make for stock test questions that allow teachers to fool their students into the belief that they have learned something. [4].

It is easy to tell what topics editors of reformed textbooks find nonessential by looking at the number of pages devoted to the discussion of that topic. For example, in The Harvard Calculus there is exactly one section on limits. In Ostebee and Zorn's text, there are two sections on limits, and in James Stewart's *Calculus: Early Transcendentals* [13], considered to be one of the most traditionally written textbooks, there are five sections that deal with limits.

In the Harvard Calculus section on limits, a limit is defined, and a graphical example is given. There is a brief (two paragraph) explanation of one and two sided limits which is followed by a short description of continuity and a definition of continuous. The reader is directed to Appendix B for an informal definition of what it means for a function to be continuous [7]. In contrast, Stewart provides a lengthy and substantial discussion of limits over the course of five sections. In these sections there are ten definitions, one of which includes the formal delta-epsilon definition of a limit. In addition, five theorems are presented including the Squeeze Theorem. There are numerous graphical and non-graphical examples provided to supplement the definitions [13].

In general, this distinction of broad and general versus specific and in depth discussion continues throughout the Harvard and Stewart texts respectively. This seems a little surprising since reformed Calculus argues that topics are cut from the curriculum in order to gain more in depth study of the remaining topics. The Harvard text does have a far greater number of 'real life' examples than the Stewart book. Almost every explanation and stated example deals with an application from economics, physics or medicine. There are applications in the Stewart text, but rather than being used throughout the book, there are chapters that target the applications of differentiation and integration.

5 Review of Studies on Reformed Calculus

With the creation of the various reform curriculums the need for evaluation of effectiveness has arisen. The National Science Foundation has been one of the driving forces of evaluative studies by providing funding to schools to conduct such studies. With over 500 universities implementing reformed Calculus in some form there have been numerous studies conducted. The results of these studies have been as wide and varied as the types of reformed Calculus implemented. The following reviews of studies from University of Illinois at Chicago, Oklahoma State University and Roger Williams University are representative of the spectrum of results.

University of Illinois at Chicago (UIC)

The study at UIC was conducted by Judith Baxter, Dibyen Majumdar and Stephen Smith [3]. The goal of the study was to compare success of traditional and reform students in subsequent math-based courses. The student pool for traditional Calculus came from students who started in the Calculus sequence in 1994-1995 and the pool for reformed Calculus came from students who started the Calculus sequence in 1995-1996. Traditional or reform, respectively, was the only method taught these academic years. Thus, there was no choice involved for students in course selection.

Students' grades in Calculus I, the Math-ACT, and the university math placement exam were compared among the traditional and reform students. The sample sizes for each method were almost identical. The results show that although the math-ACT of students in the traditional section was slightly higher, only 579 students passed Calculus I (52%) whereas 730 reform students passed reformed Calculus I (64%).

The professors at UIC determined that the best way to evaluate the success of reformed Calculus would be to track the students' success in subsequent courses. The large number of Calculus I students continuing with Physics I (science and engineering tracks) made Physics I a clear choice for making this comparison. The results showed that reformed Calculus students performed significantly better in physics than traditional students with respect to the course grade average (3.210 vs 2.646). A further breakdown of grades shows that 78% of reform students made a grade of C or better in physics while only 59% of traditional students earned a C or better. In addition to Physics I, this study also examined grade success in other subsequent math-based courses (C Programming, Organic Chemistry, Calculus II, Physics II and Calculus III). The results show that reform students also earned statistically significant higher grades in all but Physics II and Calculus III, where traditional students outperformed reform students.

From the results of the UIC study, several conclusions were drawn. First, reformed Calculus results in better grades for students than traditional Calculus. Second, the trend of higher grades continues for reform students into the courses that are taken immediately following the reformed Calculus instruction. This grade advantage does disappear in courses that are taken longer after the initial Calculus course, as can be seen by the lower scores in Physics II and Calculus III. Baxter, Majumdar and Smith attribute this dip in scores to difficulty in transitioning from more reform minded classes back to traditional curriculum.

Oklahoma State University (OSU)

OSU began implementation of a reformed Calculus curriculum in the fall of 1992. This study [8] used data from that fall until the spring semester of 1994. The OSU mathematics department opted to use the Harvard reform curriculum in addition to the traditional material as well as offering the Harvard curriculum for both Calculus I and II. The study set out to answer four questions: Do Harvard students make better grades in Calculus than traditional students? Are they more likely to enroll in subsequent math courses? Do they perform better in those classes? and How do they perform when switching from Harvard Calculus I to Standard Calculus II?

After collecting data from all students who were enrolled in a Calculus course between 1992 and 1994, their results show that Harvard Calculus students do in fact make better grades than their traditional counterparts. The difference in scores is significant with 67% of Harvard Calculus students making a grade of C or better in Calculus I and only 62% of traditional students earning a C or higher. The grade gap became even larger in the Calculus II course with 80% of Harvard students and 71% of traditional Calculus students earning a grade of C or higher.

The question of whether or not Harvard Calculus students were more likely to enroll in additional mathematics courses proved to be more difficult to determine. The answer most likely depends more on the student's major than the form of Calculus taken. Engineering and science majors are required to take math courses beyond Calculus I, while some students may only need to take Calculus I. Taking this into account, the data showed that enrollment in subsequent mathematics courses is similar for Harvard and Traditional students. One interesting statistic was that of the students that continued with Calculus II, 44% of Harvard Calculus I students switched to traditional Calculus II, while only 18% of traditional Calculus I students made the switch to Harvard for Calculus II. This statistic is not helpful in answering the question posed, but it would be interesting in studying why so many reform students decided to switch to traditional Calculus for their second semester.

Unlike the UIC study, the research at OSU showed that reform students did not perform as well as traditional students in mathematics courses after Calculus I. Of the students continuing to take Calculus II, 45% of students that made a D or better in the Harvard Calculus I were able to maintain the same grade or better in the second semester of Calculus, whereas 53% of traditional students were able to do the same. It is important to note that some of the drop in the percentage for the Harvard students may be related to the fact that students switched to a traditional Calculus II course. Outside of the Calculus sequence, OSU's traditional students continued to outperform their Harvard Calculus counterparts. Based on the Calculus I scores, 69% of traditional Calculus students were able to maintain or improve their grade in linear algebra. Only 60% of Harvard students were able to retain the same grade. The study found similar results in regards to the differential equations course.

The final question of the study resulted in significant findings. When the Calculus I and Calculus II grades of students were compared across all four possible combinations of traditional and Harvard Calculus for the first and second semesters the results indicate that some combinations are better than others. For students that remained in traditional both semesters or in Harvard both semesters, a high percentage were able to earn a grade of C or better in Calculus II (81.1% and 80.6% respectively). Making the switch from traditional Calculus I to Harvard Calculus II proved to be a good decision with 92% of the students earning a grade of C or higher. By far the worst combination for students was to take Harvard Calculus I and traditional Calculus II. Only 55.3% of students were able to finish with a grade of C or higher and a small 25.5% were able to maintain or better their grade from Calculus I.

The answers to the four questions posed by the OSU mathematics department led to a slightly different conclusion than the UIC study. The study concludes that Harvard reform students do indeed outperform the traditional Calculus students, but this advantage disappears after Calculus. The traditional students scored higher in post-Calculus mathematics classes. This lower performance may be attributed to the difficulty of transitioning from a reform, application based course to a traditional, algebraically based course. So it appears that reformed Calculus, particularly the Harvard curriculum, does increase the number of students passing Calculus but it may have a negative effect on the preparation and success of students in additional mathematics courses.

Roger Williams University (RWU)

In evaluating the effectiveness of the reformed Calculus curriculum developed by Dubinsky and Schwingendorf at Purdue, the mathematics department study at RWU [11] employed two models of assessment. The quantitative assessment evaluated student performance on a common final exam given to both the traditional and reform students. Additionally, a qualitative assessment was based on observation of student discussions, observations, misconceptions, thought patterns and problem solving strategies. The qualitative segment also included student interviews conducted throughout the semester. The quantitative assessment allowed for a set of objective data while the qualitative assessment gave a window into the thoughts of the students.

The common final examination was written by professors of traditional Calculus and was designed with a focus on traditional questions. Every faculty member that taught Calculus, traditional or reformed, was involved in grading the exam in order to minimize differences in grading amongst professors. Test results over three semesters were collected and average grades were calculated for each quartile of students enrolled in a Calculus course. During the first two semesters that the common exam was given, the reform students in all quartiles scored significantly lower than the corresponding quartiles of traditional students. From the top to the bottom quartile, traditional students' average scores were A, B, C, C whereas the equivalent reform students' scores were B-, C, D and F. However, by the third semester the exam scores had evened out and the reform students were beginning to outscore the traditional students.

One of the most interesting results from the RWU study came from examining the test scores of reform and traditional students. As seen in the first two semesters and only slightly less so in the third semester, traditional Calculus was the better choice for students in the top three quartiles. But, the exact opposite was true for students in the first quartile. In the first two semesters, the average scores of students in the bottom quartile were 27% and 28%, a far cry from passing. The third semester of reform curriculum resulted in a dramatic difference for the same group of students. The average score jumped to 63% which equated to a C.

The conclusions drawn by the professors at RWU are that although traditional Calculus works for the upper quartile of students, reform curriculum may be best suited to struggling students. The more application based and less rigorous course gives students a new experience of learning Calculus. It seems that this difference in curriculum allows students to perform better in their Calculus courses.

6 Difficulties in Reformed Calculus Studies

After reviewing several studies similar to those mentioned above, it became clear that completing a study of this nature is inherently difficult. There are a number of factors that can not be controlled or manipulated to create a fair basis of comparison. Perhaps the largest uncontrolled factor is the student sample for the reform and traditional courses. In discussing the methods of the C^4L study at Purdue University North Central, Keith Schwingendorf notes, "random assignment of students to the C^4L and traditional courses would have been preferable to self selection" [10]. Random placement would reduce the number of factors that could influence the study. Such factors might be deciding which course to take based on the professor teaching or the possibility of students with strong backgrounds taking the traditional course which would provide an unbiased student sample.

Although random placement would be ideal, there are few if any universities that would direct students into courses. This would most likely lead to a new set of problems for the study. Resentment might develop between students as the semester progresses and the differences in the course format are discussed among the students. Resentment would affect the attitudes of the students and the way they approach the course, which might negatively affect the outcomes of the study. The only case that avoids the issue of placement altogether is to offer one type of course as UIC did in its study of reform. But since the point of a study is to evaluate the success of the reform curriculum, it seems a little hasty to completely switch to a reform curriculum before the study has been completed. As a result of these difficulties, self placement will remain the standard even though it introduces a greater number of confounding factors.

How a reform course is taught is also a factor that is difficult to control. There are elaborate ways of trying to account for degree of difficulty in grading and testing across professors, such as the RWU procedure of having all the Calculus professors grade a portion of the final exam. In the end, the teaching and teacher bias greatly effects the outcome of students in the course.

In reading many studies, UIC has been the only one to mention training for professors in teaching the reformed curriculum. Acknowledging the fact that teaching reformed curriculum is in fact different from teaching a traditional curriculum and that learning how to teach reform is crucial to the success of the students and the course. It appears that many professors have offered or been forced to teach a reformed Calculus course with little or no preparation. In order for the reform curriculum to be effective it must be presented in the way it was designed to be taught. Many reform professor feel strongly about the teaching of the reformed curriculum. Michael Reed notes,

> ... teaching Reform Calculus requires more talent and training. The Old Calculus could be delivered by Professor Hohum, Mr. Part-time...because lecturing on technique is easy. Getting them to deliver Reform Calculus will be much more difficult. [2].

It is possible to limit the effect of teaching. Providing training for the professors and/or teaching assistants that will be teaching reformed Calculus will decrease some of the discrepancies. The approach of UIC provides an excellent example. The training session provides an introduction to complexities of using graphing calculators as well as in depth "explanations of the interactive teaching methods required in the reformed Calculus" [3]. In addition the UIC mathematics department required that all teaching assistants take a semester long course on the intricacies of the interactive teaching methods and group work typical of reformed Calculus courses before being assigned to the Calculus courses. The last step in providing training would be to make it mandatory for all those who would be teaching a reformed course. Training will not be effective if only a portion of the professors attend.

Teaching bias is a factor that is more difficult to minimize. Even worse than lack of training are situations in which professors are made to teach a reform course even though they may not believe in the pedagogical approach of reform. If a professor has a negative attitude toward the curriculum, then that negativity will permeate the way that the material is presented and ultimately will not engage the students. There is a seemingly obvious solution to eliminate this factor, simply to let professors who support reform teach the reformed sections. In an ideal situation this would be the perfect solution. But with the large numbers of students enrolled in Calculus, the few professors that support reform would be overtaxed with teaching all of the Calculus courses. Thus, professor attitude toward the reform material will continue to be one of the confounding factors in the experimental results.

Closely linked to professor bias, student bias is an additional element that is difficult to account for. Several studies have noted that initially students were reluctant to accept new curriculum, and hence had a negative view of reformed Calculus. This negative attitude also had a negative effect on student's performance. As more high schools are adopting a reformed Calculus curriculum, a student's initial reluctant bias could quite possibly become less of an issue, which would a help to lower the impact of student bias on studies.

Testing bias may be the hardest factor to account for. One of the easiest ways to compare student achievement across two different courses is to compare performance on a common test. But can a common test fairly assess the knowledge of reform and traditional students? The common final examination given at RWU was written by traditional Calculus teachers with a traditional emphasis. Would that be a fair judge of what the reform students know since through the course of the semester their course had a different emphasis and method of assessment? There are schools that have taken a slightly different approach. At some universities, mathematics professors that are involved in the Calculus sequence write the exam and try to balance traditional and reform questions. But again, can we expect traditional students to perform well on a reform type question to which they have never been exposed or vice versa? Despite the flaws of the common final exam, it will remain an integral part of studies because currently there is no better alternative for providing numerical data.

7 Experimental Design

The three studies reviewed above all approached the evaluation of reformed Calculus with different methods of evaluation and arrived at three different conclusions. UIC found that reformed Calculus is the best choice for students in Calculus and for students taking subsequent mathematics courses. OSU concluded that reformed Calculus benefits students while they are in the course but leaves them unprepared for future courses. RWU determined that a traditional course is most beneficial for students with a strong math background while reformed Calculus is best for students that have struggled with math previously. These results are representative of the spectrum of results from numerous studies conducted by various university mathematics departments across the country.

With so many studies employing different methods of evaluation and releasing varied results, it is difficult to compare one study to another or to compare several studies to each other. In addition to different methods of evaluation, there are few schools that actually use the same reformed curriculum. Even if the same base reformed curriculum is used at several schools, each individual school tailors the curriculum to work best within their setting. This issue of varied implementation will remain, but if a standard method of evaluation were to be adopted by the universities it would allow for a better comparison between schools. A standard evaluation would also increase the validity of the results that are presented.

The following are suggested elements for an experimental protocol. If a study such as this became a standard method of evaluation that was implemented by multiple universities, it would allow for an increase in the comparison between results from different universities.

Elements of Study:

- Randomize students. In an ideal setting, students would not be opposed to being placed in a specific course. We would assign students to a reformed or traditional course making sure to secure a mixed sample in each course (equal numbers of high and low SAT/ACT scores etc).

- Mandatory teacher training. This would allow professors to be adequately prepared to teach the reformed as well as traditional curriculum.

- Account for differences in teaching style in both the traditional and reformed courses. Have a randomly assigned rotation of teachers through both sections of Calculus. Each teacher would be responsible for teaching the same portion of curriculum to both a reformed and traditional Calculus class. This requirement would ensure that the only change in teaching style would be the method with which the curriculum is presented. In addition, each teacher would write the portion of the test which he or she taught.

- Poll students at the beginning of the course with the question: Do you think or know that you will take another math course? Then at the end of the semester ask: Will you or would you consider taking another math course? These questions will show how the Calculus course has affected the students' opinion of math.

- Administer a common final exam with questions representative of both curriculums. The common final exam is a direct way of quantitatively comparing the knowledge learned by students in both the reform and traditional courses.

- Track students to see how many from each section take additional math or math-based courses.

- Track grades of students in subsequent math courses.
- Account for different grading scales by having teachers report the scores of students (i.e. A,B,C,D,F) as well as the number of students receiving A's, B's, C's, D's and F's. This will remove the teacher bias from scoring because the numbers will show if an earning an A was a significant achievement or if a majority of the class was able to earn an A.

It is hoped that a study based on these elements would control for some of the confounding factors and biases discussed in the previous sections. With the exception of assigning students to specific courses, the author believes that the other elements are realistic options in the university setting.

Having a standard evaluation is one of the keys to allowing for comparison across universities. The other key is to have studies report the same data. Based on the author's research, the following are suggestions for the data that should be reported from studies on the effectiveness of reformed Calculus.

Data and Results:

- Provide average SAT/ACT scores of students entering the reform and traditional course sections.
- Provide the average test results from the common final exam. Report averages by quartile.
- Provide the final average grades by quartile.
- Provide the results from the survey question from the beginning and the end of the semester. This will give a clearer picture as too how each course changes the opinion of students on whether or not they will continue taking math or math related courses.
- Provide the number of students that take an additional math or math-based course as well as their grade in that course.
- Conduct student interviews with students that do continue to determine their transition experience from Calculus I to the subsequent math course. The response to these interviews will provides picture of how reformed Calculus students adjust to the traditionally taught upper-division courses.

Having data reported from the same categories will benefit research in several ways. This will allow for direct comparison of information from different schools and different variations of reformed curriculum. It will also decrease the possibility of universities only reporting the data that will support a certain position.

8 Conclusions

The opinions and study results on the effectiveness of reformed Calculus have been numerous, but from these wide ranging results there seems to be several general opinions established. First, reformed Calculus appears to be beneficial for students with a weak mathematics background. The restructured curriculum of reformed Calculus with a strong emphasis on applications to the real world gives struggling students an opportunity to see that Calculus is in fact useful and important.

This trend can be seen by the increase in average grades and passing rates of reformed Calculus students.

Although reformed Calculus may increase grades, a second trend to emerge from the studies shows that reformed Calculus leaves students unprepared for traditionally taught post-Calculus math courses. Despite the fact that reformed Calculus emphasizes written explanations of underlying thought processes, students are not exposed to the technical mathematical language that is necessary in upper-division math courses. Han Sah, an advanced Calculus professor at SUNY Stony Brook, comments on the level of preparedness of students in his class that had taken reformed Calculus I, saying "students coming in from the reformed Calculus, I have to fill in all the details and missing topics" [14].

Finally, even though the differences in curriculum are great, the author feels that a great deal of the success or failure of reformed or traditional Calculus lies in the hands of the professor. If a professor believes in and is excited about teaching a reformed curriculum, then his or her students will have a greater chance of succeeding. The same holds for a professor of traditional Calculus. George Andrews, a math professor at University of Pennsylvania, notes that

> The single most important factor in an individual's education is his teachers...not buildings, programs, curricula, philosophies of education but men and women who...showed us things we had not seen before" [2].

Hopefully in the future these opinions will be proven to be true, or possibly false, as the study of reformed Calculus continues to develop. With the development and implementation of a standard method of evaluation and distribution of result data, the author feels that the mathematics community will be able to concretely see the effectiveness, or ineffectiveness, of reformed Calculus when compared with traditional Calculus.

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