A Guide to Presentations in LaTeX-beamer

with a detour to Geometric Analysis

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Outline

Outline

1. Intro to a Presentation
2. Intro to LaTeX

Outline

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Slide Presentations

What you should not do...

- In a slide presentation, you should not list too much information in a single slide. This causes the audience to spend too much time reading the slide and not paying any attention on what you are saying. You should never just read of the slide, otherwise why not just give a handout and leave? It is also `nice` to use "fancy" *letters* and fonts as well as clever coloring schemes but they can be hard to read. Also, make sure you turn yourself towards the audience. Generally, the first part is to be understood by everyone, then you can begin to be more specific. It is common to only have the attention of a few "experts" by the end of the presentation. This applies mostly to research talks, but also to Senior Project presentations.

Some Symbols

LaTeX is a mathematics typesetting program.

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- (M^2, g) $\leftrightarrow$ `$(M^2,g)$`
- $\Delta u - K(x) - e^{2u} = 0$ $\leftrightarrow$ `$\Delta u -K(x) - e^{2u} = 0$`
- $$\inf_{n\in\mathbb{N}}\left\{\dfrac{1}{n}\right\}=0$$

  `$\ds\inf_{n\in\mathbb{N}}\set{\dfrac{1}{n}}=0$`

More Examples
without displaystyle

- $\sum_{n=1}^{\infty} \frac{1}{n^2} = \dfrac{\pi^2}{6}$

More Examples

without displaystyle

- $\sum_{n=1}^{\infty}\frac{1}{n^2} = \dfrac{\pi^2}{6}$
- `$\sum_{n=1}^{\infty}\frac{1}{n^2} = \dfrac{\pi^2}{6}$`

More Examples
with displaystyle

- $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$

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Compare displaystyle

- $\sum_{n=1}^{\infty} \frac{1}{n^2} = \dfrac{\pi^2}{6}$ versus $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

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  `and`

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Common functions

- $\cos x$ ⟶ `$\cos x$`

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Common functions

- $\cos x$ → `$\cos x$`
- $\arctan x$ → `$\arctan x$`
- $f(x) = \sqrt{x^2+1}$ → `$f(x) = \sqrt{x^2+1}$`

Common functions

- $\cos x \longrightarrow$ `$\cos x$`
- $\arctan x \longrightarrow$ `$\arctan x$`
- $f(x) = \sqrt{x^2+1} \longrightarrow$ `$f(x) = \sqrt{x^2+1}$`
- $f(x) = \sqrt[n]{x^2+1} \longrightarrow$ `$f(x) = \sqrt[n]{x^2+1}$`

Theorems

Theorem (Poincaré Inequality)

If $|\Omega| < \infty$, *then*

$$\lambda_1(\Omega) = \inf_{u \neq 0} \frac{|\nabla u|_2^2}{\|u\|^2} > 0$$

is achieved.

Theorems - code

Theorem (Poincaré Inequality)

If $|\Omega| < \infty$, then

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is achieved.

```
\begin{thm}[Poincar\'{e} Inequality]
If $|\Omega| < \infty$, then
\[
 \lambda_1(\Omega) =
 \inf_{u\neq 0} \dfrac{|\nabla u|^2_2}{\|u\|^2} > 0
\]
is achieved.
\end{thm}
```

Example - Arrays

- $$\left\{\begin{array}{ccc} -\Delta u + \lambda u & = & |u|^{p-2}, & \text{in } \Omega \\ u & \geq & 0, & u \in H_0^1(\Omega) \end{array}\right.$$

Example - Arrays

- $\left\{ \begin{array}{cccc} -\Delta u +\lambda u &= & |u|^{p-2}, &\textrm{ in } \Omega \\ u &\geq & 0, & u\in H_0^1(\Omega) \end{array} \right.$

- ```
  $\left\{
    \begin{array}{cccc}
    -\Delta u +\lambda u &= & |u|^{p-2}, &\textrm{ in }
    \Omega \\
    u &\geq & 0, & u\in H_0^1(\Omega)
    \end{array}
  \right.$
  ```
  ```

Example - Arrays
Change centering

- $$\left\{ \begin{array}{lcr} -\Delta u + \lambda u & = & |u|^{p-2}, \quad \text{in } \Omega \\ u & \geq & 0, \quad u \in H_0^1(\Omega) \end{array} \right.$$

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- ```
  $\left\{
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    \Omega \\
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    \end{array}
  \right.$
  ```
  ```

More Examples

- $\varphi(u) = \int_\Omega \left[\frac{\|\nabla u\|^2}{2} + \lambda \frac{u^2}{2} - \frac{(u^+)^p}{p} \right] d\mu$

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- $\varphi(u) = \displaystyle\int_{\Omega} \left[\dfrac{\|\nabla u\|^2}{2} + \lambda\dfrac{u^2}{2} - \dfrac{(u^+)^p}{p} \right] d\mu$

- ```
  $\ds \varphi (u) = \int_{\Omega} \left[
  \dfrac{\|\nabla u\|^2}{2} +
  \lambda\dfrac{u^2}{2} -
  \dfrac{(u^+)^p}{p} \right] d\mu $
  ```
  ```

Even More Examples

De Morgan's Law

- $\left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^{n} A_i^c$

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- $$\left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$$
- `$\ds \left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$`

Even More Examples

De Morgan's Law

- $\left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$
- `$\ds \left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$`
- $A \times B = \{(a, b) | a \in A, b \in B\}$

Even More Examples

De Morgan's Law

- $\displaystyle \left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$

- `$\ds \left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$`

- $A \times B = \{(a,b)|a\in A, b\in B\}$

- `$A\times B = \set{(a,b)|a\in A, b\in B}$`

Equations

- Consider the equation of Energy below.

$$E(u) = \int |\nabla u|^2 dx \tag{1}$$

This is how we refer to (1).

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- ```
  \begin{equation}\label{eq:energy}
      E(u) = \int |\nabla u|^2 dx
  \end{equation}
  ```

  ```
  This is how we refer to \eqref{eq:energy}.
  ```
  ```

Equations

- Consider the equation without a number below.

$$E(u) = \int |\nabla u|^2 dx$$

Equations

- Consider the equation without a number below.

$$E(u) = \int |\nabla u|^2 dx$$

- ```
  \begin{equation}\label{eq:energy}
      E(u) = \int |\nabla u|^2 dx \nonumber
  \end{equation}
  ```
  ```

Equations

Tag an equation

- Consider the equation with a tag

$$E(u) = \int |\nabla u|^2 dx \tag{E}$$

If u is harmonic, (E) is preserved.

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Tag an equation

- Consider the equation with a tag

$$E(u) = \int |\nabla u|^2 dx \tag{E}$$

If u is harmonic, (E) is preserved.

- ```
  \begin{equation}\label{eq:energytag}
      E(u) = \int |\nabla u|^2 dx \tag{E}
  \end{equation}

  If $u$ is harmonic, \eqref{eq:energytag} is preserved.
  ```
  ```

Equations

a small proof

- Indeed,

$$\begin{aligned} \frac{d}{dt}E(u) &= 2\int \langle \nabla u, \nabla u \rangle \\ &= -2\int \langle \Delta u, u \rangle = 0 \end{aligned} \tag{2}$$

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- Indeed,

$$\begin{aligned} \frac{d}{dt}E(u) &= 2\int \langle \nabla u, \nabla u \rangle \\ &= -2\int \langle \Delta u, u \rangle = 0 \end{aligned} \tag{2}$$

- The "=" signs are aligned.

Equations

a small proof

- Indeed,

$$\begin{split} \frac{d}{dt}E(u) &= 2\int \langle \nabla u, \nabla u \rangle \\ &= -2\int \langle \Delta u, u \rangle = 0 \end{split} \tag{2}$$

- The "=" signs are aligned.
- Use

  `\begin{split} ... \end{split}`

Equations

a small proof

- ```
  \begin{equation}
   \begin{split}
    \dfrac{d}{dt} E(u) & = 2 \int \langle\nabla u,
                                \nabla u\rangle\\
    & = -2 \int\langle\Delta u,u\rangle = 0
   \end{split}
  \end{equation}
  ```
  ```

Equations

in an array

- Consider the expression below

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+2ab+b^2\end{aligned} \tag{3}$$

Equations
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$$\begin{split}(a+b)^2 &= (a+b)(a+b) \\ &= a^2 + 2ab + b^2\end{split} \tag{3}$$

- ```
  \begin{equation}
   \begin{split}
    (a+b)^2 &   = (a+b)(a+b) \\
            &   = a^2 +2ab +b^2
   \end{split}
  \end{equation}
  ```
  ```

Environments

In LaTeX, environments must match:

- ```
  \begin{...}
  .
  .
  .
  \end{...}
  ```
  ```

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- $ ...$ → for math symbols
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In LaTeX, environments must match:

- ```
  \begin{...}
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  .
  .
  \end{...}
  ```

- `$ ...$` → for math symbols
- `\[ ... \]` → for centering expressions
  ```

Environments

In LaTeX, environments must match:

- ```
  \begin{...}
  .
  .
  .
  \end{...}
  ```
- $ ...$ → for math symbols
- `\[ ... \]` → for centering expressions
- `\left( ... \right)` → match size of parentheses
  ```

Environments
delimiters

- $(\int |\nabla u|^p d\mu)^p$ versus $\left(\int |\nabla u|^p d\mu\right)^p$

Environments

delimiters

- $(\displaystyle\int|\nabla u|^p d\mu)^p$ versus $\left(\displaystyle\int|\nabla u|^p d\mu\right)^p$
- `$(\ds\int|\nabla u|^p d\mu)^p$`
- `$\left(\ds\int|\nabla u|^p d\mu\right)^p$`

Tables

Consider the truth table:

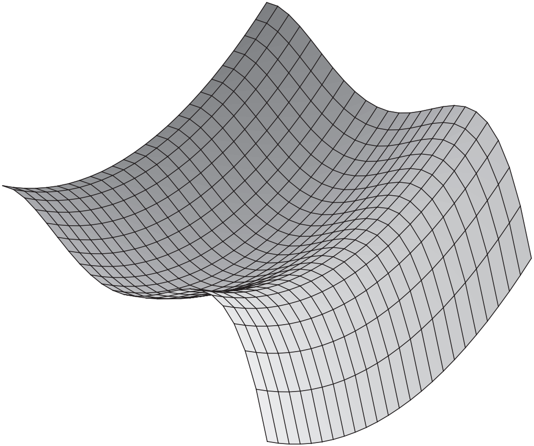
P	Q	$\neg P$	$\neg P \rightarrow (P \vee Q)$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Tables - code

```
  \begin{tabular}{c c c | c}
$P$ & $Q$ & $\neg P$ & $\neg P\to (P \vee Q)$ \\ \hline
T & T & F & T \\
T & F & F & T \\
F & T & T & T \\
F & F & T & F

\end{tabular}
```

Inserting Pictures
Mountain Pass Landscape



Inserting Pictures - code

```
\begin{center}
  \includegraphics{Mountain_Pass.eps}
\end{center}
```

Inserting Pictures

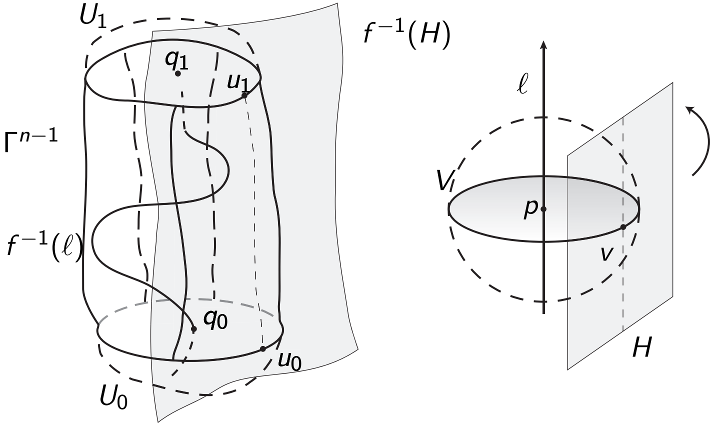


Figure: Construction of Γ^n by revolving affine hyperplanes

A Final Remark on LaTeX

Preamble

- Preamble → "Stuff" on top of .tex file

```
%For an article using AMS template:
\documentclass[12pt]{amsart}
\usepackage{amsmath,amssymb,amsfonts,amsthm}
...
```

A Final Remark on LaTeX

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- Don't worry about it!

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%For an article using AMS template:
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- Don't worry about it!

- With practice you can figure it out.

How a Slide is done in Beamer

my subtitle

This is a slide

- First Item
- Second Item

How a Slide is done in Beamer

my subtitle

The code should look like:

```
\begin{frame}

  \frametitle{How a Slide is done in Beamer}
  \framesubtitle{my subtitle}  % optional
This is a slide
  \begin{itemize}
    \item First Item
    \item Second Item
  \end{itemize}

\end{frame}
```

How a Slide is done in Beamer

In this document...

The option [fragile] is added to use verbatim

```
\begin{verbatim}
Anything you type $\Delta u = |u|^{p-2}$

Appears as is in the code

end{verbatim}

Note: if you correctly type \ end{verbatim} it assumes
      verbatim environment ended.

Hence ``fragile''
```

How a Slide with pause is done in Beamer

This is a slide

- First Item

How a Slide with pause is done in Beamer

This is a slide

- First Item
- Second Item

How a Slide with pause is done in Beamer

The code should look like:

```
\begin{frame}

  \frametitle{How a Slide with pause is done in Beamer}
This is a slide
  \begin{itemize}
    \item First Item
    \pause
    \item Second Item
  \end{itemize}

\end{frame}
```

Overlay example

- First item

- Fourth item

Overlay example

- First item
- Second item
- Fourth item

Overlay example

- First item
- Second item
- Third item
- Fourth item

Overlay example

The code should look like:

```
\begin{frame}[fragile]
  \frametitle{Overlay example}

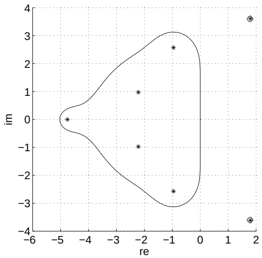
  \begin{itemize}
    \only<1->{\item First item}
    \uncover<2->{\item Second item}
    \uncover<3->{\item Third item}
    \only<1->{\item Fourth item}
  \end{itemize}

\end{frame}
```

Example of Figures

Taken from online Template.

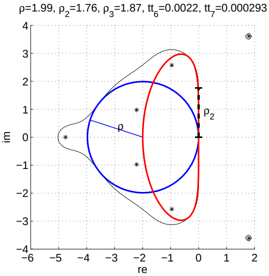
- Using `\only<1>` to draw the first two Figures



Example of Figures

Taken from online Template.

- Using \only<1> to draw the first two Figures
- Using \only<2-> to draw the second two Figures



Need a plain slide?

Add [plain] option to the slide.

Variational Calculus

A simple Idea to solve equations:

- Solve $f(x) = 0$

Variational Calculus

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- Suppose we know that $F' = f$.

Variational Calculus

A simple Idea to solve equations:

- Solve $f(x) = 0$
- Suppose we know that $F' = f$.
- Critical points of F are solutions of $f(x) = 0$.

Variational Calculus

An idea from Calculus I:

Variational Calculus

An idea from Calculus I:

Theorem (Rolle)

Let $f \in C^1([x_1, x_2]; \mathbb{R})$. If $f(x_1) = f(x_2)$, then there exists $x_3 \in (x_1, x_2)$ such that $f'(x_3) = 0$.

Variational Calculus

An idea from Calculus I:

Theorem (Rolle)

Let $f \in C^1([x_1, x_2]; \mathbb{R})$*. If* $f(x_1) = f(x_2)$*, then there exists* $x_3 \in (x_1, x_2)$ *such that* $f'(x_3) = 0$*.*

```
\begin{thm}[Rolle]
Let $f\in C^1([x_1,x_2];\mathbb{R})$. If $f(x_1)=f(x_2)$,
then there exists $x_3\in(x_1,x_2)$
such that $f'(x_3) = 0$.
\end{thm}
```

Variational Calculus

Rolle's Theorem has the following landscape.

Variational Calculus

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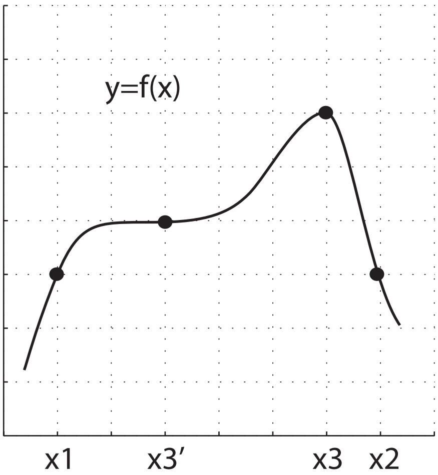


Variational Calculus - Code

```
\begin{frame}
  \frametitle{Variational Calculus}
  \uncover<1->{
  Rolle's Theorem has the following landscape.
  }
  \uncover<2->{\begin{center}
  \includegraphics{rolle.eps}
  \end{center}
  }

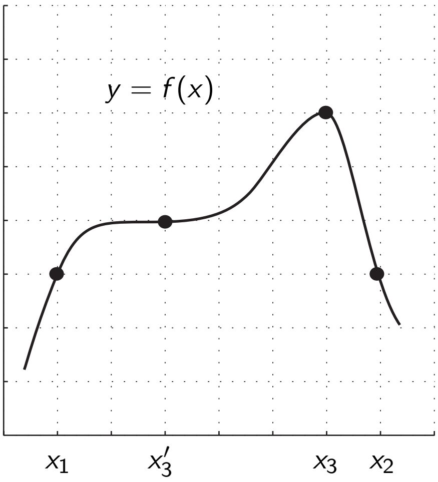
\end{frame}
```

Variational Calculus - psfrags

Rolle's Theorem has the following landscape.

Variational Calculus - psfrags

Rolle's Theorem has the following landscape.

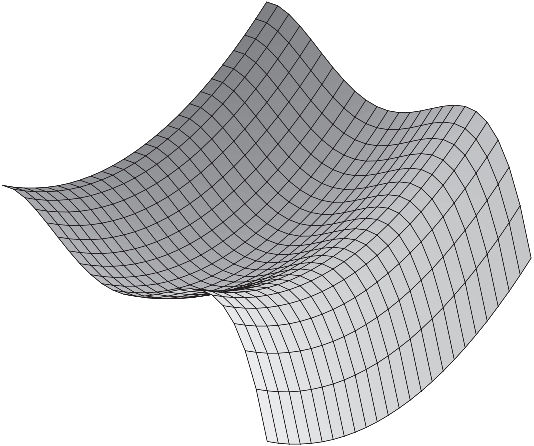


Variational Calculus - psfrags - Code

```
\begin{frame}
  \frametitle{Variational Calculus - psfrags}
  \uncover<1->{Rolle's Theorem has the following landscape
  \uncover<2->{\begin{figure}[h]
\begin{center}
\begin{psfrags}
\psfrag{x1}{$x_1$}\psfrag{x2}{$x_2$}
\psfrag{x3}{$x_3$}\psfrag{x3'}{$x_3'$}
\psfrag{y=f(x)}{$y=f(x)$}
\includegraphics{rolle.eps}
\end{psfrags}
\end{center}
\end{figure}
  }

\end{frame}
```

Mountain Pass Landscape



MPT - presentation

Not a friendly introduction

Theorem (Finite Dimensional MPT, Courant)

Suppose that $\varphi \in C^1(\mathbb{R}^n, \mathbb{R})$ *is coercive and possesses two distinct strict relative minima* x_1 *and* x_2*. Then* φ *possesses a third critical point* x_3 *distinct from* x_1 *and* x_2*, characterized by*

$$\varphi(x_3) = \inf_{\Sigma \in \Gamma} \max_{x \in \Sigma} \varphi(x)$$

where $\Gamma = \{\Sigma \subset \mathbb{R}^n; \Sigma \text{ is compact and connected and } x_1, x_2 \in \Sigma\}$*. Moreover,* x_3 *is not a relative minimizer, that it, in every neighborhood of* x_3 *there exists a point* x *such that* $\varphi(x) < \varphi(x_3)$*.*

MPT - in parts

- When displaying large info do in steps.

MPT - in parts

- When displaying large info do in steps.
- Avoid audience reading ahead and not paying attention.

MPT - presentation
A friendly introduction

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MPT - presentation
A friendly introduction

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MPT - presentation

A friendly introduction

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$$\varphi(x_3) = \inf_{\Sigma \in \Gamma} \max_{x \in \Sigma} \varphi(x)$$

MPT - presentation

A friendly introduction

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$\Gamma = \{\Sigma \subset \mathbb{R}^n; \Sigma \text{ is compact and connected and } x_1, x_2 \in \Sigma\}$.

MPT - presentation

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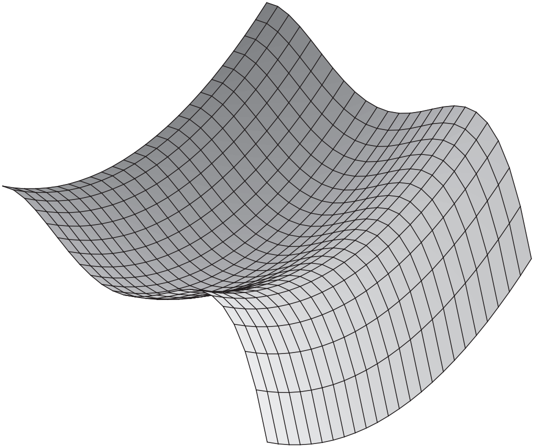
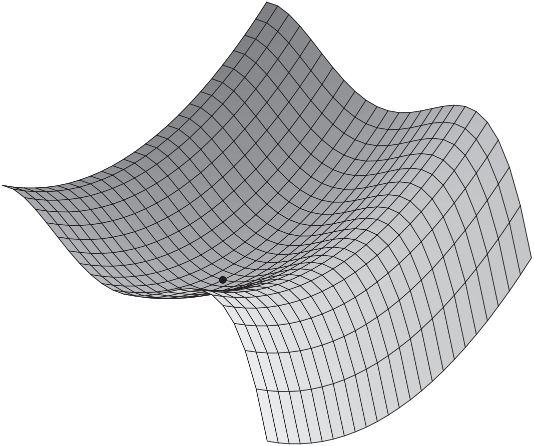
$$\varphi(x_3) = \inf_{\Sigma \in \Gamma} \max_{x \in \Sigma} \varphi(x)$$

where
$\Gamma = \{\Sigma \subset \mathbb{R}^n; \Sigma$ is compact and connected and $x_1, x_2 \in \Sigma\}$.
Moreover, x_3 is not a relative minimizer, that it, in every neighborhood of x_3 there exists a point x such that $\varphi(x) < \varphi(x_3)$.

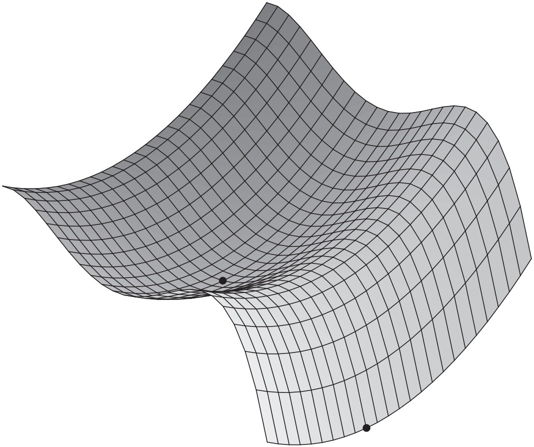
Geometry of MPT



Geometry of MPT



Geometry of MPT



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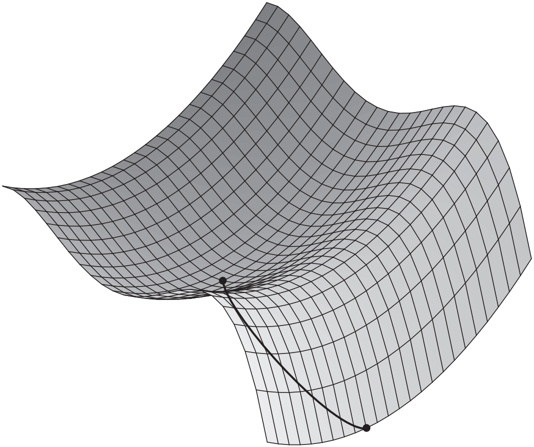

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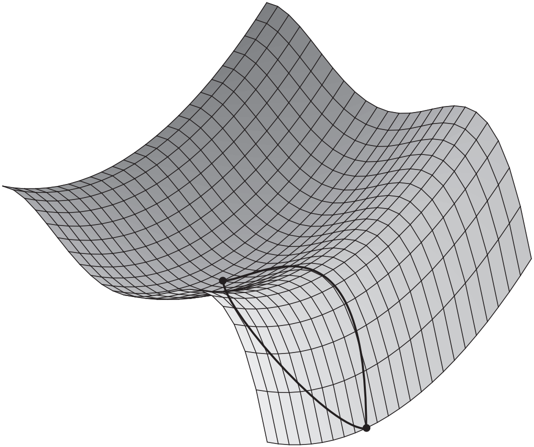

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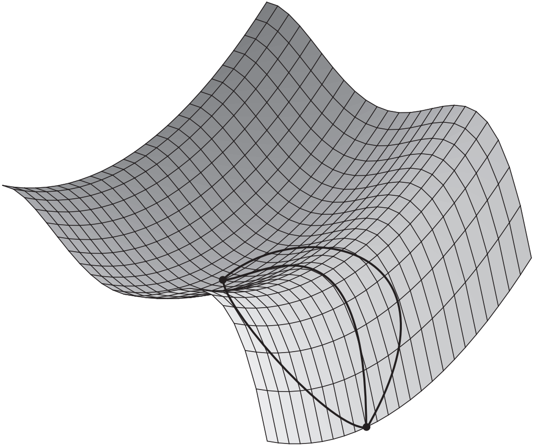

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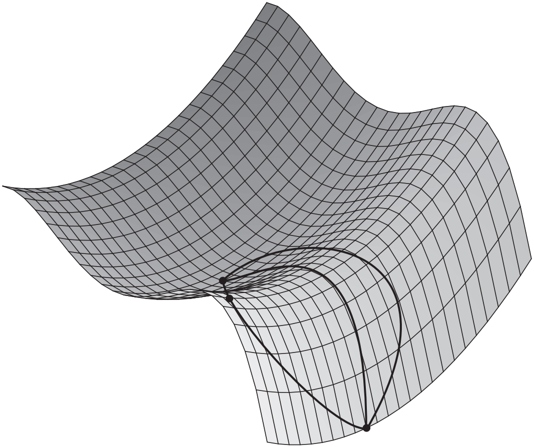

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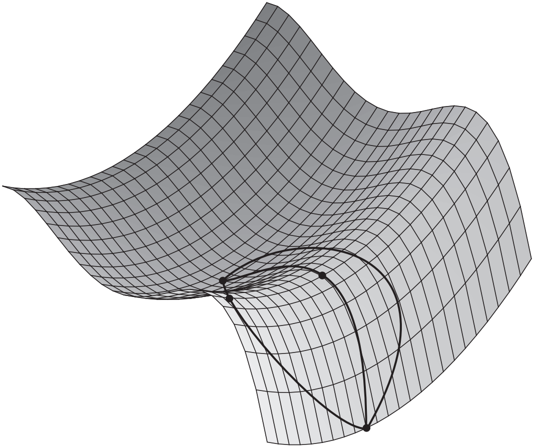

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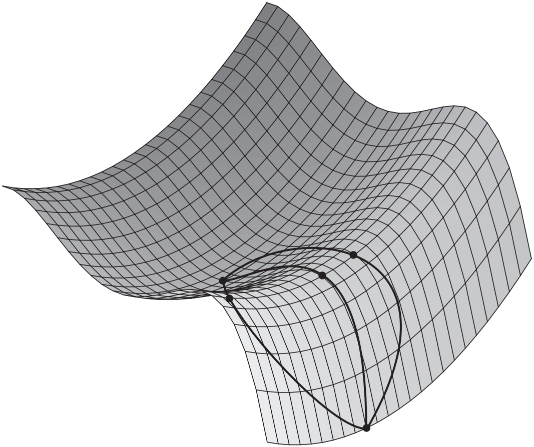

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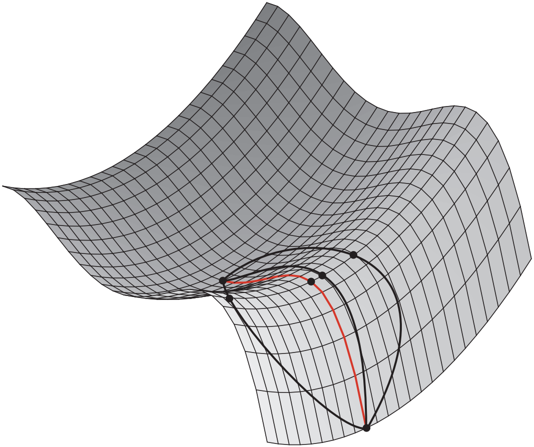

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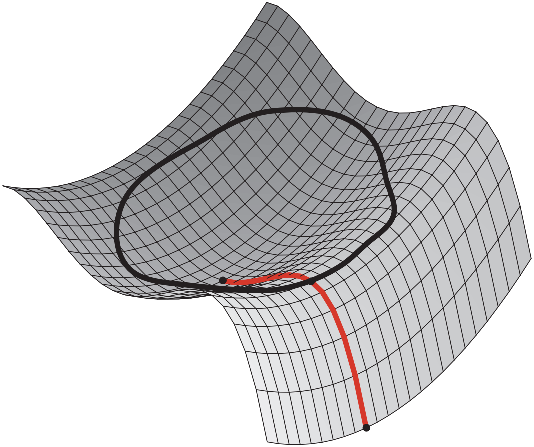

An Application of MPT

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- You only need the idea.
- The actual proof will be in your final paper!

An Application of MPT

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Let X and Y be finite dimensional Euclidean spaces, and let $\varphi : X \to Y$ be a C^1 function such that:

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dimensional Euclidean spaces, and let
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            }
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            }
\uncover<4->{\noindent Then $\varphi$ is a
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 \end{thm}
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