A Guide to Presentations in LaTeX-beamer

with a detour to Geometric Analysis

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Outline

Some Symbols

LaTeX is a mathematics typesetting program.

- Standard Language to Write Mathematics
- $(M^2, g) \leftrightarrow$ `$(M^2,g)$`
- $\Delta u - K(x) - e^{2u} = 0 \leftrightarrow$ `$\Delta u -K(x) - e^{2u} = 0$`
- $\displaystyle\inf_{n\in\mathbb{N}} \left\{\dfrac{1}{n}\right\} = 0$

  `$\ds\inf_{n\in\mathbb{N}}\set{\dfrac{1}{n}}=0$`

Compare displaystyle

- $\sum_{n=1}^{\infty}\frac{1}{n^2}=\dfrac{\pi^2}{6}$ versus $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$
- `$\sum_{n=1}^{\infty}\frac{1}{n^2}=\dfrac{\pi^2}{6}$`

  and

  `$\ds\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$`

Common functions

- $\cos x$ → `$\cos x$`
- $\arctan x$ → `$\arctan x$`
- $f(x) = \sqrt{x^2+1}$ → `$f(x) = \sqrt{x^2+1}$`
- $f(x) = \sqrt[n]{x^2+1}$ → `$f(x) = \sqrt[n]{x^2+1}$`

Theorems - code

Theorem (Poincaré Inequality)

If $|\Omega| < \infty$, then

$$\lambda_1(\Omega) = \inf_{u\neq 0} \dfrac{|\nabla u|^2_2}{\|u\|^2} > 0$$

is achieved.

```
\begin{thm}[Poincar\'{e} Inequality]
If $|\Omega| < \infty$, then
\[
 \lambda_1(\Omega) =
 \inf_{u\neq 0} \dfrac{|\nabla u|^2_2}{\|u\|^2} > 0
\]
is achieved.
\end{thm}
```

Example - Arrays

- $\left\{ \begin{array}{cccc} -\Delta u + \lambda u &= & |u|^{p-2}, &\textrm{ in } \Omega \\ u &\geq & 0, & u\in H_0^1(\Omega) \end{array} \right.$

- ```
  $\left\{
    \begin{array}{cccc}
    -\Delta u +\lambda u &= & |u|^{p-2}, &\textrm{ in }
    \Omega \\
    u &\geq & 0, & u\in H_0^1(\Omega)
    \end{array}
  \right.$
  ```
  ```

Example - Arrays

Change centering

- $\left\{ \begin{array}{lcrr} -\Delta u + \lambda u &= & |u|^{p-2}, & \textrm{ in } \Omega \\ u &\geq & 0, & u\in H_0^1(\Omega) \end{array} \right.$

- ```
  $\left\{
    \begin{array}{lcrr}
    -\Delta u +\lambda u &= & |u|^{p-2}, &\textrm{ in }
    \Omega \\
    u &\geq & 0, & u\in H_0^1(\Omega)
    \end{array}
  \right.$
  ```
  ```

Example - Arrays

Change centering

- $\left\{ \begin{array}{rcll} -\Delta u + \lambda u &= & |u|^{p-2}, &\textrm{ in } \Omega \\ u &\geq & 0, & u\in H_0^1(\Omega) \end{array} \right.$

- ```
  $\left\{
    \begin{array}{rcll}
    -\Delta u +\lambda u &= & |u|^{p-2}, &\textrm{ in }
    \Omega \\
    u &\geq & 0, & u\in H_0^1(\Omega)
    \end{array}
  \right.$
  ```
  ```

More Examples

- $\displaystyle \varphi (u) = \int_{\Omega} \left[\dfrac{\|\nabla u\|^2}{2} + \lambda\dfrac{u^2}{2} - \dfrac{(u^+)^p}{p} \right] d\mu$

- ```
  $\ds \varphi (u) = \int_{\Omega} \left[
  \dfrac{\|\nabla u\|^2}{2} +
  \lambda\dfrac{u^2}{2} -
  \dfrac{(u^+)^p}{p} \right] d\mu $
  ```
  ```

Even More Examples

De Morgan's Law

- $\displaystyle \left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$
- `$\ds \left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^n A_i^c$`
- $A \times B = \{(a,b)|a\in A, b\in B\}$
- `$A\times B = \set{(a,b)|a\in A, b\in B}$`

Equations

- Consider the equation of Energy below.

$$E(u) = \int |\nabla u|^2 dx \tag{1}$$

  This is how we refer to (1).

-
```
\begin{equation}\label{eq:energy}
    E(u) = \int |\nabla u|^2 dx
\end{equation}

This is how we refer to \eqref{eq:energy}.
```

Equations

- Consider the equation without a number below.

$$E(u) = \int |\nabla u|^2 dx$$

- ```
  \begin{equation}\label{eq:energy}
      E(u) = \int |\nabla u|^2 dx \nonumber
  \end{equation}
  ```
  ```

Equations
Tag an equation

- Consider the equation with a tag

$$E(u) = \int |\nabla u|^2 dx \tag{E}$$

If u is harmonic, (E) is preserved.

- ```
  \begin{equation}\label{eq:energytag}
      E(u) = \int |\nabla u|^2 dx \tag{E}
  \end{equation}
  ```

  ```
  If $u$ is harmonic, \eqref{eq:energytag} is preserved.
  ```
  ```

Equations

in an array

- Consider the expression below

$$\begin{split} (a+b)^2 &= (a+b)(a+b) \\ &= a^2 + 2ab + b^2 \end{split} \tag{2}$$

- ```
  \begin{equation}
   \begin{split}
    (a+b)^2 &  = (a+b)(a+b) \\
            &  = a^2 +2ab +b^2
   \end{split}
  \end{equation}
  ```
  ```

Environments

In LaTeX, environments must match:

- ```
  \begin{...}
  .
  .
  .
  \end{...}
  ```
- $ ...$ → for math symbols
- `\[ ... \]` → for centering expressions
- `\left( ... \right)` → match size of parentheses
  ```

Environments
delimiters

- $(\displaystyle\int|\nabla u|^p d\mu)^p$ versus $\left(\displaystyle\int|\nabla u|^p d\mu\right)^p$

- `$(\ds\int|\nabla u|^p d\mu)^p$`
- `$\left(\ds\int|\nabla u|^p d\mu\right)^p$`

Tables

Consider the truth table:

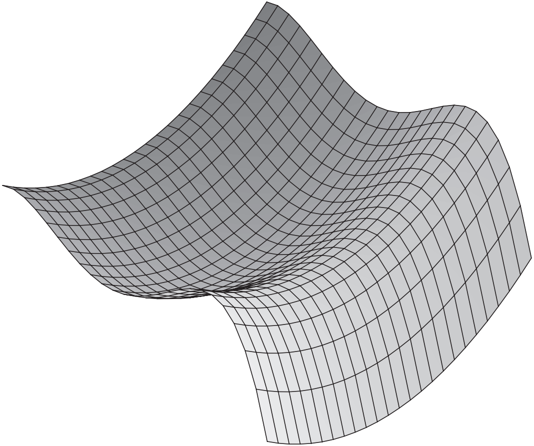
P	Q	$\neg P$	$\neg P \to (P \vee Q)$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Tables - code

```
  \begin{tabular}{c c c | c}
$P$ & $Q$ & $\neg P$ & $\neg P\to (P \vee Q)$ \\ \hline
T & T & F & T \\
T & F & F & T \\
F & T & T & T \\
F & F & T & F

\end{tabular}
```

Inserting Pictures
Mountain Pass Landscape



Inserting Pictures - code

```
\begin{center}
  \includegraphics{Mountain_Pass.eps}
\end{center}
```

Inserting Pictures

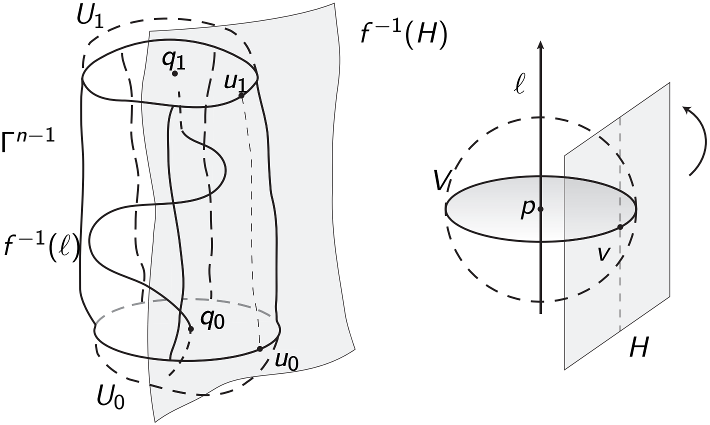


Figure: Construction of Γ^n by revolving affine hyperplanes

A Final Remark on LaTeX
Preamble

- Preamble → "Stuff" on top of .tex file

```
%For an article using AMS template:
\documentclass[12pt]{amsart}
\usepackage{amsmath,amssymb,amsfonts,amsthm}
...
```

- Don't worry about it!

- With practice you can figure it out.

How a Slide is done in Beamer

my subtitle

This is a slide

- First Item
- Second Item

How a Slide is done in Beamer

my subtitle

The code should look like:

```
\begin{frame}

  \frametitle{How a Slide is done in Beamer}
  \framesubtitle{my subtitle}  % optional
This is a slide
  \begin{itemize}
    \item First Item
    \item Second Item
  \end{itemize}

\end{frame}
```

How a Slide with pause is done in Beamer

This is a slide

- First Item
- Second Item

How a Slide with pause is done in Beamer

The code should look like:

```
\begin{frame}

  \frametitle{How a Slide with pause is done in Beamer}
This is a slide
  \begin{itemize}
    \item First Item
    \pause
    \item Second Item
  \end{itemize}

\end{frame}
```

Overlay example

- First item
- Second item
- Third item
- Fourth item

Overlay example

The code should look like:

```
\begin{frame}[fragile]
  \frametitle{Overlay example}

  \begin{itemize}
    \only<1->{\item First item}
    \uncover<2->{\item Second item}
    \uncover<3->{\item Third item}
    \only<1->{\item Fourth item}
  \end{itemize}

\end{frame}
```

Need a plain slide?

Add [plain] option to the slide.

Variational Calculus

A simple Idea to solve equations:

- Solve $f(x) = 0$
- Suppose we know that $F' = f$.
- Critical points of F are solutions of $f(x) = 0$.

Variational Calculus

An idea from Calculus I:

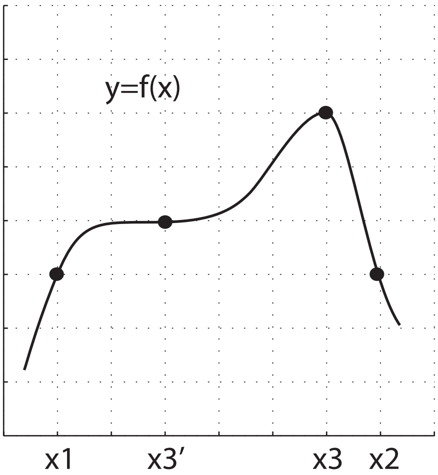
Theorem (Rolle)

Let $f \in C^1([x_1, x_2]; \mathbb{R})$. *If* $f(x_1) = f(x_2)$, *then there exists* $x_3 \in (x_1, x_2)$ *such that* $f'(x_3) = 0$.

```
\begin{thm}[Rolle]
Let $f\in C^1([x_1,x_2];\mathbb{R})$. If $f(x_1)=f(x_2)$,
then there exists $x_3\in(x_1,x_2)$
such that $f'(x_3) = 0$.
\end{thm}
```

Variational Calculus

Rolle's Theorem has the following landscape.



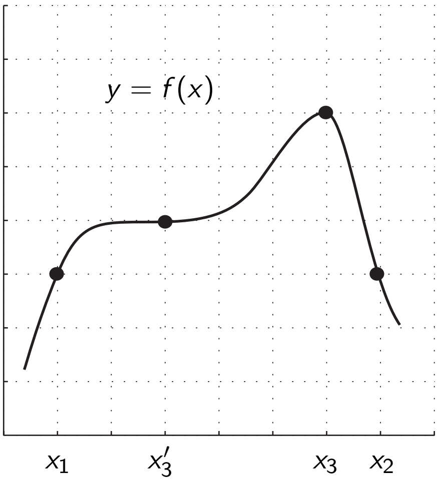
Variational Calculus - Code

```
\begin{frame}
  \frametitle{Variational Calculus}
  \uncover<1->{
  Rolle's Theorem has the following landscape.
  }
  \uncover<2->{\begin{center}
  \includegraphics{rolle.eps}
  \end{center}
  }

\end{frame}
```

Variational Calculus - psfrags

Rolle's Theorem has the following landscape.



Variational Calculus - psfrags - Code

```
\begin{frame}
  \frametitle{Variational Calculus - psfrags}
  \uncover<1->{Rolle's Theorem has the following landscape
  \uncover<2->{\begin{figure}[h]
\begin{center}
\begin{psfrags}
\psfrag{x1}{$x_1$}\psfrag{x2}{$x_2$}
\psfrag{x3}{$x_3$}\psfrag{x3'}{$x_3'$}
\psfrag{y=f(x)}{$y=f(x)$}
\includegraphics{rolle.eps}
\end{psfrags}
\end{center}
\end{figure}
  }

\end{frame}
```

MPT - presentation
A friendly introduction

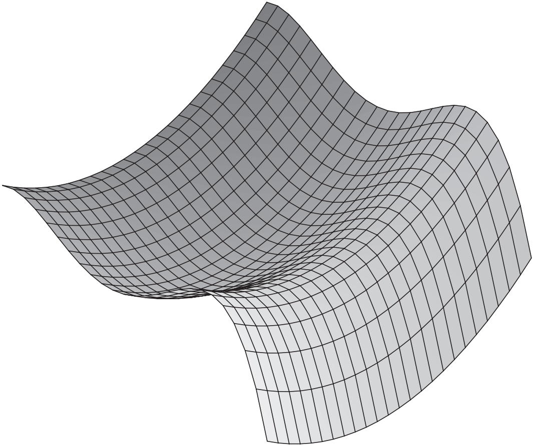
Theorem (Finite Dimensional MPT, Courant)

Suppose that $\varphi \in C^1(\mathbb{R}^n, \mathbb{R})$ is coercive and possesses two distinct strict relative minima x_1 and x_2. Then φ possesses a third critical point x_3 distinct from x_1 and x_2, characterized by

$$\varphi(x_3) = \inf_{\Sigma \in \Gamma} \max_{x \in \Sigma} \varphi(x)$$

where
$\Gamma = \{\Sigma \subset \mathbb{R}^n; \Sigma$ *is compact and connected and* $x_1, x_2 \in \Sigma\}$.
Moreover, x_3 is not a relative minimizer, that it, in every neighborhood of x_3 there exists a point x such that $\varphi(x) < \varphi(x_3)$.

Mountain Pass Landscape



An Application of MPT

Theorem (Hadamard)

Let X and Y be finite dimensional Euclidean spaces, and let $\varphi : X \to Y$ be a C^1 function such that:

(i) *$\varphi'(x)$ is invertible for all $x \in X$.*

(ii) *$\|\varphi(x)\| \to \infty$ as $\|x\| \to \infty$.*

Then φ is a diffeomorphism of X onto Y.

An Application of MPT
Hadamard's Theorem - Idea of Proof

- Check that φ is onto.
- Prove injectivity by contradiction.
- Suppose $\varphi(x_1) = \varphi(x_2) = y$, then define

$$f(x) = \frac{1}{2} \|\varphi(x) - y\|^2$$

- Check the MPT geometry for f.
- $\exists x_3$, $f(x_3) > 0$ (i.e., $\|\varphi(x_3) - y\| > 0$.)
- $f'(x_3) = \nabla^T \varphi(x_3) \cdot (\varphi(x_3) - y) = 0$