

A Guide to Presentations in \LaTeX -beamer with a detour to Geometric Analysis

Eduardo Balreira



Trinity University
Mathematics Department

Major Seminar, Fall 2008

Outline

1 Intro to \LaTeX

Outline

- 1 Intro to \LaTeX
- 2 Intro to Beamer

Outline

- 1 Intro to \LaTeX
- 2 Intro to Beamer
- 3 Geometric Analysis

Outline

- 1 Intro to \LaTeX
- 2 Intro to Beamer
- 3 Geometric Analysis
- 4 A Proof

Some Symbols

LaTeX is a mathematics typesetting program.

- Standard Language to Write Mathematics

Some Symbols

LaTeX is a mathematics typesetting program.

- Standard Language to Write Mathematics
- $(M^2, g) \leftrightarrow \$(M^2, g)\$$

Some Symbols

LaTeX is a mathematics typesetting program.

- Standard Language to Write Mathematics

- $(M^2, g) \leftrightarrow \$(M^2, g)\$$

- $\Delta u - K(x) - e^{2u} = 0 \leftrightarrow \$\Delta u - K(x) - e^{2u} = 0\$$

Some Symbols

LaTeX is a mathematics typesetting program.

- Standard Language to Write Mathematics

- $(M^2, g) \leftrightarrow \$(M^2, g)\$$

- $\Delta u - K(x) - e^{2u} = 0 \leftrightarrow \$\Delta u - K(x) - e^{\{2u\}} = 0\$$

- $\inf_{n \in \mathbb{N}} \left\{ \frac{1}{n} \right\} = 0$

$$\$\ds\inf_{\{n \in \mathbb{N}\}} \set{\dfrac{1}{n}}=0\$$$

Compare displaystyle

- $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ versus $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

Compare displaystyle

- $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ versus $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

- $\$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}\$$

and

$$\$\ds\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}\$$$

Common functions

- $\cos x \rightarrow \text{\code{\cos x}}$

Common functions

- $\cos x \rightarrow \text{\code{\cos x}}$
- $\arctan x \rightarrow \text{\code{\arctan x}}$

Common functions

- $\cos x \rightarrow \text{\code{\cos x}}$
- $\arctan x \rightarrow \text{\code{\arctan x}}$
- $f(x) = \sqrt{x^2 + 1} \rightarrow \text{\code{f(x) = \sqrt{x^2+1}}}$

Common functions

- $\cos x \rightarrow \text{\code{\cos x}}$
- $\arctan x \rightarrow \text{\code{\arctan x}}$
- $f(x) = \sqrt{x^2 + 1} \rightarrow \text{\code{f(x) = \sqrt{x^2+1}}}$
- $f(x) = \sqrt[n]{x^2 + 1} \rightarrow \text{\code{f(x) = \sqrt[n]{x^2+1}}}$

Theorems - code

Theorem (Poincaré Inequality)

If $|\Omega| < \infty$, then

$$\lambda_1(\Omega) = \inf_{u \neq 0} \frac{|\nabla u|_2^2}{\|u\|_2^2} > 0$$

is achieved.

```
\begin{thm}[Poincar\'{e} Inequality]
```

```
If  $|\Omega| < \infty$ , then
```

```
\[
```

```
\lambda_1(\Omega) =
```

```
\inf_{u \neq 0} \frac{|\nabla u|_2^2}{\|u\|_2^2} > 0
```

```
\]
```

```
is achieved.
```

```
\end{thm}
```


Example - Arrays

Change centering

- $$\begin{cases} -\Delta u + \lambda u = |u|^{p-2}, & \text{in } \Omega \\ u \geq 0, & u \in H_0^1(\Omega) \end{cases}$$

Example - Arrays

Change centering

- $$\begin{cases} -\Delta u + \lambda u = |u|^{p-2}, & \text{in } \Omega \\ u \geq 0, & u \in H_0^1(\Omega) \end{cases}$$

- ```

$$\left\{ \begin{array}{rcll} -\Delta u + \lambda u & = & |u|^{p-2}, & \text{\texttrm{ in } } \\ \Omega & \\ u & \geq & 0, & u \in H_0^1(\Omega) \end{array} \right.$$


```

# More Examples

- $$\varphi(u) = \int_{\Omega} \left[ \frac{\|\nabla u\|^2}{2} + \lambda \frac{u^2}{2} - \frac{(u^+)^p}{p} \right] d\mu$$

# More Examples

- $\varphi(u) = \int_{\Omega} \left[ \frac{\|\nabla u\|^2}{2} + \lambda \frac{u^2}{2} - \frac{(u^+)^p}{p} \right] d\mu$
- $\text{\$}\backslash ds \ \text{\$} \varphi(u) = \int_{\Omega} \left[ \frac{\|\nabla u\|^2}{2} + \lambda \frac{u^2}{2} - \frac{(u^+)^p}{p} \right] d\mu \text{\$}$

# Even More Examples

De Morgan's Law

- $$\left( \bigcup_{i=1}^n A_i \right)^c = \bigcap_{i=1}^n A_i^c$$

# Even More Examples

De Morgan's Law

- $\left(\bigcup_{i=1}^n A_i\right)^c = \bigcap_{i=1}^n A_i^c$

- $\left(\bigcup_{i=1}^n A_i\right)^c = \bigcap_{i=1}^n A_i^c$

# Even More Examples

De Morgan's Law

- $\left(\bigcup_{i=1}^n A_i\right)^c = \bigcap_{i=1}^n A_i^c$

- $\left(\bigcup_{i=1}^n A_i\right)^c = \bigcap_{i=1}^n A_i^c$

- $A \times B = \{(a, b) | a \in A, b \in B\}$

# Even More Examples

De Morgan's Law

- $\left(\bigcup_{i=1}^n A_i\right)^c = \bigcap_{i=1}^n A_i^c$
- $\left(\bigcup_{i=1}^n A_i\right)^c = \bigcap_{i=1}^n A_i^c$
- $A \times B = \{(a, b) \mid a \in A, b \in B\}$
- $A \times B = \set{(a,b) \mid a \in A, b \in B}$



# Equations

- Consider the equation of Energy below.

$$E(u) = \int |\nabla u|^2 dx \quad (1)$$

This is how we refer to (1).

# Equations

- Consider the equation of Energy below.

$$E(u) = \int |\nabla u|^2 dx \tag{1}$$

This is how we refer to (1).

- ```
\begin{equation}\label{eq:energy}
E(u) = \int |\nabla u|^2 dx
\end{equation}
```

This is how we refer to `\eqref{eq:energy}`.

Equations

- Consider the equation without a number below.

$$E(u) = \int |\nabla u|^2 dx$$

Equations

- Consider the equation without a number below.

$$E(u) = \int |\nabla u|^2 dx$$

- `\begin{equation}\label{eq:energy}`
 `E(u) = \int |\nabla u|^2 dx \nonumber`
`\end{equation}`

Equations

Tag an equation

- Consider the equation with a tag

$$E(u) = \int |\nabla u|^2 dx \tag{E}$$

If u is harmonic, (E) is preserved.

Equations

Tag an equation

- Consider the equation with a tag

$$E(u) = \int |\nabla u|^2 dx \tag{E}$$

If u is harmonic, (E) is preserved.

- ```
\begin{equation}\label{eq:energytag}
 E(u) = \int |\nabla u|^2 dx \tag{E}
\end{equation}
```

If  $u$  is harmonic, `\eqref{eq:energytag}` is preserved.

# Equations

## a small proof

- ```
\begin{equation}
\begin{split}
\frac{d}{dt} E(u) &= 2 \int \langle \nabla u,
& \nabla u \rangle \\
&= -2 \int \langle \Delta u, u \rangle = 0
\end{split}
\end{equation}
```

Equations

in an array

- Consider the expression below

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a^2 + 2ab + b^2\end{aligned}\tag{2}$$

Equations

in an array

- Consider the expression below

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a^2 + 2ab + b^2\end{aligned}\tag{2}$$

- ```
\begin{equation}
\begin{split}
(a+b)^2 &= (a+b)(a+b) \\
&= a^2 + 2ab + b^2
\end{split}
\end{equation}
```

# Environments

In  $\LaTeX$ , environments must match:

- $\backslash\text{begin}\{\dots\}$ 
  - 
  - 
  -
- $\backslash\text{end}\{\dots\}$

# Environments

In  $\LaTeX$ , environments must match:

- $\backslash\text{begin}\{\dots\}$ 
  - 
  - 
  -
- $\backslash\text{end}\{\dots\}$
- $\$ \dots \$ \rightarrow$  for math symbols

# Environments

In  $\LaTeX$ , environments must match:

- $\backslash\text{begin}\{\dots\}$   
•  
•  
•  
•  $\backslash\text{end}\{\dots\}$
- $\$ \dots \$$   $\rightarrow$  for math symbols
- $\backslash[ \dots \backslash]$   $\rightarrow$  for centering expressions

# Environments

In LaTeX, environments must match:

- `\begin{...}`  
  .  
  .  
  .  
  `\end{...}`
- `$ ... $` → for math symbols
- `\[ ... \]` → for centering expressions
- `\left( ... \right)` → match size of parentheses

# Environments

## delimiters

- $\left(\int |\nabla u|^p d\mu\right)^p$  versus  $\left(\int |\nabla u|^p d\mu\right)^p$

# Environments

## delimiters

- $\left(\int |\nabla u|^p d\mu\right)^p$  versus  $\left(\int |\nabla u|^p d\mu\right)^p$
- $\$(\ds\int|\nabla u|^p d\mu)^p$$
- $\$\left(\ds\int|\nabla u|^p d\mu\right)^p$$

# Tables

Consider the truth table:

| $P$ | $Q$ | $\neg P$ | $\neg P \rightarrow (P \vee Q)$ |
|-----|-----|----------|---------------------------------|
| T   | T   | F        | T                               |
| T   | F   | F        | T                               |
| F   | T   | T        | T                               |
| F   | F   | T        | F                               |



## Tables - code

```
\begin{tabular}{c c c | c}
 P & Q & $\neg P$ & $\neg P \rightarrow (P \vee Q)$ \\ \hline
T & T & F & T \\
T & F & F & T \\
F & T & T & T \\
F & F & T & F
\end{tabular}
```

# Inserting Pictures - code

```
\begin{center}
 \includegraphics{Mountain_Pass.eps}
\end{center}
```

## Inserting Pictures - code - psfrags - Code

```
\begin{frame}
 \frametitle{Variational Calculus - psfrags}
 \uncover<1->{Rolle's Theorem has the following landscape}
 \uncover<2->{\begin{figure}[h]
\begin{center}
\begin{psfrags}
\psfrag{x1}{{x_1}}\psfrag{x2}{{x_2}}
\psfrag{x3}{{x_3}}\psfrag{x3'}{{x_3'}}
\psfrag{y=f(x)}{{$y=f(x)$}}
\includegraphics{rolle.eps}
\end{psfrags}
\end{center}
\end{figure}
}

\end{frame}
```

# How a Slide is done in Beamer

my subtitle

This is a slide

- First Item
  
- Second Item

# How a Slide is done in Beamer

my subtitle

The code should look like:

```
\begin{frame}

 \frametitle{How a Slide is done in Beamer}
 \framesubtitle{my subtitle} % optional
 This is a slide
 \begin{itemize}
 \item First Item
 \item Second Item
 \end{itemize}

\end{frame}
```

# How a Slide with pause is done in Beamer

This is a slide

- First Item

# How a Slide with pause is done in Beamer

This is a slide

- First Item
- Second Item

# How a Slide with pause is done in Beamer

The code should look like:

```
\begin{frame}

 \frametitle{How a Slide with pause is done in Beamer}
 This is a slide
 \begin{itemize}
 \item First Item
 \pause
 \item Second Item
 \end{itemize}

\end{frame}
```



# Overlay example

- First item

- Fourth item

# Overlay example

- First item
- Second item
- Fourth item

# Overlay example

- First item
- Second item
- Third item
- Fourth item

# Overlay example

The code should look like:

```
\begin{frame}[fragile]
 \frametitle{Overlay example}

 \begin{itemize}
 \only<1->{\item First item}
 \uncover<2->{\item Second item}
 \uncover<3->{\item Third item}
 \only<1->{\item Fourth item}
 \end{itemize}

\end{frame}
```

Need a plain slide?

Add [plain] option to the slide.

# Variational Calculus

A simple Idea to solve equations:

- Solve  $f(x) = 0$

# Variational Calculus

A simple Idea to solve equations:

- Solve  $f(x) = 0$
  
- Suppose we know that  $F' = f$ .

# Variational Calculus

A simple Idea to solve equations:

- Solve  $f(x) = 0$
- Suppose we know that  $F' = f$ .
- Critical points of  $F$  are solutions of  $f(x) = 0$ .



# Variational Calculus

An idea from Calculus I:

## Theorem (Rolle)

*Let  $f \in C^1([x_1, x_2]; \mathbb{R})$ . If  $f(x_1) = f(x_2)$ , then there exists  $x_3 \in (x_1, x_2)$  such that  $f'(x_3) = 0$ .*

```
\begin{thm}[Rolle]
```

```
Let $f \in C^1([x_1, x_2]; \mathbb{R})$. If $f(x_1) = f(x_2)$,
then there exists $x_3 \in (x_1, x_2)$
such that $f'(x_3) = 0$.
```

```
\end{thm}
```

# MPT - presentation

## A friendly introduction

### Theorem (Finite Dimensional MPT, Courant)

*Suppose that  $\varphi \in C^1(\mathbb{R}^n, \mathbb{R})$  is coercive and possesses two distinct strict relative minima  $x_1$  and  $x_2$ .*

# MPT - presentation

## A friendly introduction

### Theorem (Finite Dimensional MPT, Courant)

*Suppose that  $\varphi \in C^1(\mathbb{R}^n, \mathbb{R})$  is coercive and possesses two distinct strict relative minima  $x_1$  and  $x_2$ . Then  $\varphi$  possesses a third critical point  $x_3$  distinct from  $x_1$  and  $x_2$*

# MPT - presentation

## A friendly introduction

### Theorem (Finite Dimensional MPT, Courant)

*Suppose that  $\varphi \in C^1(\mathbb{R}^n, \mathbb{R})$  is coercive and possesses two distinct strict relative minima  $x_1$  and  $x_2$ . Then  $\varphi$  possesses a third critical point  $x_3$  distinct from  $x_1$  and  $x_2$ , characterized by*

$$\varphi(x_3) = \inf_{\Sigma \in \Gamma} \max_{x \in \Sigma} \varphi(x)$$

# MPT - presentation

## A friendly introduction

### Theorem (Finite Dimensional MPT, Courant)

*Suppose that  $\varphi \in C^1(\mathbb{R}^n, \mathbb{R})$  is coercive and possesses two distinct strict relative minima  $x_1$  and  $x_2$ . Then  $\varphi$  possesses a third critical point  $x_3$  distinct from  $x_1$  and  $x_2$ , characterized by*

$$\varphi(x_3) = \inf_{\Sigma \in \Gamma} \max_{x \in \Sigma} \varphi(x)$$

*where*

$\Gamma = \{\Sigma \subset \mathbb{R}^n; \Sigma \text{ is compact and connected and } x_1, x_2 \in \Sigma\}$ .

# MPT - presentation

## A friendly introduction

### Theorem (Finite Dimensional MPT, Courant)

*Suppose that  $\varphi \in C^1(\mathbb{R}^n, \mathbb{R})$  is coercive and possesses two distinct strict relative minima  $x_1$  and  $x_2$ . Then  $\varphi$  possesses a third critical point  $x_3$  distinct from  $x_1$  and  $x_2$ , characterized by*

$$\varphi(x_3) = \inf_{\Sigma \in \Gamma} \max_{x \in \Sigma} \varphi(x)$$

*where*

$\Gamma = \{\Sigma \subset \mathbb{R}^n; \Sigma \text{ is compact and connected and } x_1, x_2 \in \Sigma\}$ .

*Moreover,  $x_3$  is not a relative minimizer, that is, in every neighborhood of  $x_3$  there exists a point  $x$  such that  $\varphi(x) < \varphi(x_3)$ .*

# An Application of MPT

## Theorem (Hadamard)

*Let  $X$  and  $Y$  be finite dimensional Euclidean spaces, and let  $\varphi : X \rightarrow Y$  be a  $C^1$  function such that:*

# An Application of MPT

## Theorem (Hadamard)

Let  $X$  and  $Y$  be finite dimensional Euclidean spaces, and let  $\varphi : X \rightarrow Y$  be a  $C^1$  function such that:

**(i)**  $\varphi'(x)$  is invertible for all  $x \in X$ .



# An Application of MPT

## Theorem (Hadamard)

Let  $X$  and  $Y$  be finite dimensional Euclidean spaces, and let  $\varphi : X \rightarrow Y$  be a  $C^1$  function such that:

- (i)  $\varphi'(x)$  is invertible for all  $x \in X$ .
- (ii)  $\|\varphi(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ .

# An Application of MPT

## Theorem (Hadamard)

Let  $X$  and  $Y$  be finite dimensional Euclidean spaces, and let  $\varphi : X \rightarrow Y$  be a  $C^1$  function such that:

- (i)  $\varphi'(x)$  is invertible for all  $x \in X$ .
- (ii)  $\|\varphi(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ .

Then  $\varphi$  is a diffeomorphism of  $X$  onto  $Y$ .

# An Application of MPT

## Hadamard's Theorem - Idea of Proof

- Check that  $\varphi$  is onto.

# An Application of MPT

## Hadamard's Theorem - Idea of Proof

- Check that  $\varphi$  is onto.
- Prove injectivity by contradiction.

# An Application of MPT

## Hadamard's Theorem - Idea of Proof

- Check that  $\varphi$  is onto.
- Prove injectivity by contradiction.
- Suppose  $\varphi(x_1) = \varphi(x_2) = y$ , then define

$$f(x) = \frac{1}{2} \|\varphi(x) - y\|^2$$

# An Application of MPT

## Hadamard's Theorem - Idea of Proof

- Check that  $\varphi$  is onto.
- Prove injectivity by contradiction.
- Suppose  $\varphi(x_1) = \varphi(x_2) = y$ , then define

$$f(x) = \frac{1}{2} \|\varphi(x) - y\|^2$$

- Check the MPT geometry for  $f$ .

# An Application of MPT

## Hadamard's Theorem - Idea of Proof

- Check that  $\varphi$  is onto.
- Prove injectivity by contradiction.
- Suppose  $\varphi(x_1) = \varphi(x_2) = y$ , then define

$$f(x) = \frac{1}{2} \|\varphi(x) - y\|^2$$

- Check the MPT geometry for  $f$ .
- $\exists x_3, f(x_3) > 0$  (i.e.,  $\|\varphi(x_3) - y\| > 0$ .)

# An Application of MPT

## Hadamard's Theorem - Idea of Proof

- Check that  $\varphi$  is onto.
- Prove injectivity by contradiction.
- Suppose  $\varphi(x_1) = \varphi(x_2) = y$ , then define

$$f(x) = \frac{1}{2} \|\varphi(x) - y\|^2$$

- Check the MPT geometry for  $f$ .
- $\exists x_3, f(x_3) > 0$  (i.e.,  $\|\varphi(x_3) - y\| > 0$ .)
- $f'(x_3) = \nabla^T \varphi(x_3) \cdot (\varphi(x_3) - y) = 0$