Exam 2 Review Assignment
Due Date: Tuesday, March 31
30 points

1. Let \( \mathbf{u} = \langle 6, 2, 3 \rangle \) and \( \mathbf{v} = \langle 2, -3, 1 \rangle \).
   (a) Compute \( 4\mathbf{u} - 3\mathbf{v} \).
   (b) Find the length of \( \mathbf{u} \).
   (c) Find a unit vector which points in a direction opposite \( \mathbf{u} \).
   (d) Are \( \mathbf{u} \) and \( \mathbf{v} \) parallel, perpendicular, or neither? If your answer is neither, then find the angle between these vectors.
   (e) Find a vector, \( \mathbf{w} \), which is orthogonal to both \( \mathbf{u} \) and \( \mathbf{v} \).

2. Consider the points \( P(2, 5, 5) \) and \( Q(-6, 3, 1) \).
   (a) Find an equation for the line between \( P \) and \( Q \).
   (b) Find an equation of the plane consisting of all points which are equidistant from \( P \) and \( Q \).

3. Do Problem 54 in Section 12.3.

4. A differential equation representing a family of curves is given by
   \[ y' = \frac{x + y}{x}; \quad x > 0. \]
   Find the curve of this family which passes through the point \((3, 0)\).

5. Solve the initial value problem \( \frac{dy}{dx} = \frac{xy^2 + 3x}{y + x^2y}, \quad y(1) = 3. \)

6. Solve the initial value problem \( (e^y - x)\frac{dy}{dx} = 1; \quad y(1) = 0. \)
   \( \text{(Hint: } x' + P(y)x = Q(y) \text{ is a linear ODE in the function } x(y). \text{)}\)

7. Solve the initial value problem
   \[ \cos(x)\frac{dy}{dx} + \sin(x)y = \cos(x); \quad y(0) = 4; \quad \frac{-\pi}{2} < x < \frac{\pi}{2}. \]
8. Solve the initial value problem

\[ y''' - y'' + 4y' - 4y = 0; \quad y(0) = 0, y'(0) = 7, y''(0) = 9. \]

(Note that \((1)^3 - (1)^2 + 4(1) - 4 = 0\).)

9. Solve the differential equation

\[ y'' - y' = e^x + 2x. \]

10. A 5000 L tank is full of a solution. In order to clean this tank, a water solution of which 3% is liquid chlorine is pumped into the tank at a rate of 6 liters per minute, and then a mixed liquid is pumped out at a rate of 10 liters per minute.

(a) Assuming that the solution in the tank contains no chlorine when the cleaning process begins, find the amount of chlorine in the tank at any time \(t\).

(b) How much chlorine is in the tank when the tank is half full?

11. Suppose a population of koala bears, \(K\), satisfies the differential equation

\[ \frac{dK}{dt} = .2K - .000125K^2. \]

(a) If the initial population is 320 koalas, find the population at any time \(t\), where \(t\) is measured in years.

(b) Find the koala population after 6 months.

(c) How long will it take for the koala population to triple in size?

(d) Assuming this model is viable indefinitely, what will the koala population be after 62 billion years?